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Cost estimation of large construction projects with dependent risks

A study on the Brenner Base Tunnel

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Abstract

The realisation of large infrastructure projects appears as a challenging venture, not only from a technical point of view, but also in terms of methodical and effective cost evaluation. Moreover, numerous decisions throughout the phases of planning and construction, as well as during the project's service life, are firmly related to efforts to prevent, mitigate or address possible adverse events. The twofold requirement to quantify subjective perceptions of risks and to elaborate the resulting data in a mathematically consistent manner, may prove to be extremely demanding.

The scope of the present study is to investigate, compare and propose methods for efficient acquisition and utilisation of information regarding risk and cost, in long–term and large–scale tunnelling projects. The developed methods should facilitate incorporating information, or lack thereof, into the risk analysis, in order to effectively treat uncertainty and to support decision–making. Also, they should be able to assist practitioners in aggregating all underlying risks and chances, so as to to solve a complex multi–dimensional problem, such as the estimation of the total cost.

In particular, the present work focuses on the realistic representation of risk factors as individual cost elements and, consequently, on the formulation of a transparent computational setup, wherein these elements can be efficiently combined in order to yield reliable total cost estimates. Emphasis is drawn on quantitative decision–making under the—commonly encountered in singular undertakings—conditions of high uncertainty, complexity and partial information. The total cost is not viewed as a mere sum of fixed and independent values, but rather as the outcome of a stochastic framework, where risk fluctuations and dependencies are also considered. The conceptual model is inspired from and applied to the risk analysis of the Brenner Base Tunnel, one of the most important engineering projects being constructed nowadays in Europe.

Zusammenfassung

Die Realisierung großer Infrastrukturprojekte ist nicht nur aus technischer Sicht, sondern auch in Bezug auf methodische und effektive Kostenschätzungen und –Management, eine anspruchsvolle Herausforderung. Darüber hinaus sind zahlreiche Entscheidungen in der Planungs- und Ausführungsphase, sowie über die ganze Lebensdauer des Projektes hinweg, mit den Maßnahmen zur Vorbeugung, Verringerung bzw. zum Management von möglichen Risiken verbunden. Dabei können sich einerseits die Quantifizierung der subjektiven Wahrnehmungen von Risiken und andererseits die mathematisch konsistente Auswertung der daraus resultierenden Daten äußerst anspruchsvoll sein.

Die vorliegende Studie soll Methoden zur effizienten Erfassung und Verwendung von Informationen über Risiken und Kosten in Langzeit–Tunnelgroßprojekten analysieren und vergleichen, sowie neue Methoden vorschlagen. Durch die Entwicklung dieser Methoden soll erleichtert werden, die Informationen bzw. deren Mangel in die Risikoanalyse einzubauen, um Ungewissheiten effektiv anzugehen und Entscheidungsprozesse zu vereinfachen. Auf der Grundlage dieser Methoden sollen die Fachleute in die Lage versetzt werden, alle zugrundeliegenden Risiken und Chancen derart miteinander in Zusammenhang zu bringen, dass sie derart komplexe, multidimensionale Anforderungen wie die Schätzung der Gesamtkosten, strukturiert angehen können.

Insbesondere liegt das Hauptaugenmerk der vorliegenden Arbeit auf der realistischen Darstellung von Risikofaktoren, z.B. einzelne Kostenelemente, und folglich auf der Formulierung eines transparenten rechnerischen Aufbaus; dabei können diese Elemente effizient kombiniert werden, um eine verlässliche Schätzung der Gesamtkosten anstellen zu können. Der Schwerpunkt liegt auf der quantitativen Entscheidungsbildung unter unvorteilhaften Bedingungen aufgrund von Unsicherheit, Komplexität und mangelnden Informationen, welche sich insbesondere im Rahmen einzigartiger Vorhaben ergeben. Die Gesamtkosten werden nicht als einfache Summe fixer, unabhängiger Werte verstanden, sondern vielmehr als Ergebnis eines stochastischen Prozesses, in dem auch Risikoschwankungen und Abhängigkeiten berücksichtigt werden. Das konzeptionelle Modell inspiriert sich an der Risikoanalyse eines der wichtigsten Ingenieurbauprojekte, die heute in Europa in Ausführung sind, nämlich des Brenner Basistunnels.

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“Timendi causa est nescire” — *Seneka*

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Chapter 1

Introduction

1.1 Motivation and scope

The present study is motivated by the constantly increasing demand [CFR07] for reliable cost estimations in large infrastructure projects dominated by large uncertainties. In particular, the Brenner Base Tunnel[1], the most important tunnelling project under construction nowadays in Europe, serves as the basis for the theoretical investigations, as well as for the computational developments throughout the study.

The scope of the present research is to provide insight to the extent needed, with regard to the specified objectives, into the following topics:

- The concepts of risk and uncertainty (nature, sources, propagation and effect) in large–scale and long–term construction planning.
- The state of the art regarding the use of risk analysis tools in infrastructure projects, with emphasis on risk and uncertainty quantification techniques.
- The particular challenges of mega–projects, and the commonly encountered problem of cost underestimation.
- The role of experts and the importance of subjective judgement in decision–making under uncertainty.
- The probabilistic representation of risks as individual cost and time elements.
- The effect of dependence among scheduled activities and among possible events on economic estimates.

[1] A description of the project is given in Chapter 5.

- The task of risk aggregation under the conditions outlined above.
- The effective contribution to an actual ongoing infrastructure project, namely the Brenner Base Tunnel, through practical implementation of the exposed ideas and methods.

1.2 Concepts and methods

The concepts of risk and uncertainty are at the very core of modern decision–making theory and practice. The distinction and the relation between these two notions is not evident [Elk00], [PGW08]. In the present work, risks are considered from a neutral point of view, i.e. rather as possible events with negative or positive impact, instead of simply unwanted hazards affecting the project. Besides, timely and reliable information is acknowledged as a decisive factor in effectively managing risk and uncertainty. Information is generally regarded as any element of knowledge, able to modify a decision [Hub07], [Sav09], whereas the importance of its timeliness and reliability attributes is mostly case–specific.

Both risk and uncertainty possess a paradox. On the one hand, revealing more underlying risks during planning reduces the risk of the whole undertaking. This fact not only highlights the importance of timely information, but also stresses the need for a methodical risk analysis, especially when a considerable investment is at stake. On the other hand, uncertainty—even if partly viewed as ignorance—can be cognitively studied and efficiently used to supplement understanding on the processes of concern. Furthermore, uncertainty may increase with knowledge.[2]

Another ambiguous, even controversial notion[3] is that of probability. Since the present work heavily relies on probabilistic methods, the fundamental matter of probability needs to be clarified. In this respect, in practical engineering modelling at least four interpretations can be identified [GW04], [OF05], [TBF99]:

1. The *classical* interpretation, where probabilities are assigned to events through combinatorial considerations.
2. The *frequentist* interpretation, where probabilities are viewed as relative frequencies of outcomes in controlled experiments.

[2] As eloquently expressed by R.W. Sockman, *"The larger the island of knowledge, the longer the shoreline of wonder"*.

[3] Controversies regarding the theoretical foundations of probability lie beyond the scope of the present research.

3. The *possibilistic* interpretation, where probabilities are degrees of belief, subjectively attributed to events.

4. The *propensity* interpretation, where probabilities emerge as physical necessities from underlying causal mechanisms.

All four mentioned interpretations are being considered, as appropriate, throughout the following chapters. The possibilistic version results from subjective expert beliefs [FKH$^+$07], while the classical conception is required for constructing mathematically consistent models[4]. Furthermore, since probability—perceived as frequency—exists as a common viewpoint [TBF99], it may serve as a transitive mechanism in order to increase understanding, and to enable conversion among different probabilistic interpretations [FAZ09]. Finally, the propensity interpretation, further discussed in Section (4.3), reflects the inherent physical and mechanical phenomena, filtered by human causality awareness.

1.3 Outline of study

The present research is organised as follows:

In **Chapter 2**, the concept of risk and its various types encountered in large civil engineering works are discussed. Summary findings from a conducted survey on the state of the art concerning the use of quantitative risk analysis methods in such enterprises are presented. Emphasis is directed to uncertainty, differentiated from risk as hazard, and placed on a rather cognitive perspective. This shift invokes a classification of information and confidence into levels, which are strongly linked to expert judgement concepts, elicitation techniques, and modelling decisions.

Chapter 3 primarily focuses on the problem of evaluating individual cost elements. The probabilistic route is selected, closed–end densities are compared to open–end alternatives, whereas the vagueness of bounds is discussed. Then, a deeper investigation among probability distributions, typically used for uncertaint cost representation is carried out, leading to the choice of the beta density. The unimodal beta family is studied in detail, and a method developed by the author for parameter estimation based on subjective opinions is proposed. Finally, few other distributions, found in analogous studies, are cited, and a closing remark is reserved for non–probabilistic approaches.

[4] For instance, the axiomatic requirement expressed by Equation (3.1).

In **Chapter 4**, the complex problem of multivariate dependence is examined. Firstly, the complementary concepts of causation and covariation are presented in parallel. The removal of the independence assumption among random variables, which can unfold in various forms (initiating from a trivial negation), gives rise to modern sophisticated approaches for dealing with stochastic dependence. The survey on the matter evolves in the present study mainly around the targeted application, with due awareness of the limitations imposed by the scarcity of information. The requirement for plausible dependence modelling receives attention, since it can lead to feasible solutions to the problem of total cost estimation.

The Brenner Base Tunnel (BBT)—the case study of the present research—is introduced in **Chapter 5**, wherein sources for further information are also provided. Data obtained through the risk analysis of the project are presented, including assessments from experts, classification, grouping and dependencies concerning the identified risks. Practical aspects of the various risk quantification steps and relevant data acquisition efforts are demonstrated. Individual risks are modelled according to the methodology developed in Chapter 3.

The application on the multivariate problem of estimating the risk term of the total cost, is illustrated in **Chapter 6**. The techniques presented in Chapters 3 and 4 are combined with the data gathered from the actual project. Estimates are produced and evaluated on the basis of sensitivity analyses.

Finally, essential findings and generic recommendations are synopsised in the closing **Chapter 7**. Moreover, conclusions are drawn and an evaluation of the present research is attempted, in light of the numerical results.

Appendices A, **B** and **C** include the theoretical background and basic nomenclature for three substantial topics, namely statistical distributions, the Monte Carlo method, and copula functions.

Chapter 2

Risk Analysis in current practice

2.1 The concept of risk

A plethora of definitions of risk can be traced in academic studies, depending on the context and on the scholar's standpoint. For instance, Zou et al. [ZZW07] present a variety of definitions gleaned from the literature, wherein risk may be referred to as "the potential for unwanted or negative consequences of an event or activity", "a combination of hazard and exposure", or "the chance of something happening that will have a (positive or negative) impact on objectives". Elkjaer [Elk00] simply considers risk as "a normally unwanted event" and, in line with that perspective, Carr and Tah [CT01] as "a disturbance leading to a system's malfunction". An interestingly short, still powerful definition is that risk is "uncertainty that matters" [HMW07, p. 5]. In the present work, the following narrowed and explanatory position is adopted:

Risks (and opportunities[1]) are economic items, affecting the project budget, that cannot be deterministically known, since they are associated with events, decisions, processes and interrelations under conditions of randomness, vagueness and ambiguity.

A single risk R_j is typically defined in quantitative terms as the product of its probability of occurrence P_j and its impact I_j, as expressed in the following well–known Equation (2.1). This rationale exhibits at least two serious limitations [ETKV04]: it neither accounts for uncertainties of the two factors, nor it distinguishes between extreme rare and insignificant likely events. Moreover, Dikmen et al. [DBH07] argue that the magnitude of a risk is determined, not only by its likeli-

[1] In the present study, risks have a positive value in the cost account, whereas opportunities are uncertain quantities with negative sign, expressing possible revenue.

hood and impact, but also by the carrier's ability to cope with its consequences, or to influence the underlying mechanism. In any case, Equation (2.1) stands as the outset of risk evaluation:

$$R_j = P_j \cdot I_j \tag{2.1}$$

Risk *quantification*—the process aiming to produce reliable estimates for P_j and I_j—is the quantitative manifestation of an overall structured process, referred to as *risk analysis* [ETKV04]. Moreover, at that point, temporal attributes and possible dependencies among risks are defined[2] [CT01]. This methodological treatment of an infrastructure project can reveal several aspects regarding latent risks and opportunities. Furthermore, the resulting documentation enables the operator to develop the appropriate measures for preventing, mitigating and managing risks. Another advantage rests in the ability to combine the impact of all underlying risks in order to estimate the total cost and, therefore, to evaluate the feasibility of the whole project. The present study mainly concentrates on that aggregation.

A risk analysis process can obviously be performed in various degrees of detail, intensity and accuracy. The reasons for performing a thorough analysis are not limited only to the requirement for reliable total cost estimates, but additionally include the aims to [RB04], [SMJ06]:

- Reduce the risk to project goals and objectives (safety, budget, schedule, standards).
- Evaluate and justify options and decisions.
- Clarify and streamline internal goals and priorities.
- Encourage the disclosure and sharing of useful information.
- Obtain probable ranges of both cost and time.
- Assist the development of counter–measures.
- Increase confidence and reduce reliance on third parties.
- Derive scientific, formalised and reproducible methodologies.

[2] The dependence problem is tackled in Chapter 4, whereas time–related issues are not covered in the present study.

The more general term *risk management* is used to include many risk–related processes, such as[3]: planning, identification, qualification, quantification, elimination, mitigation, allocation, monitoring and control. Risk management can include the treatment of uncertainties with both negative or positive results (risk and opportunity management). The present study focuses exclusively on the risk (and opportunity) quantification step; particularly on data acquisition and elicitation, representation of (direct or indirect) economic dependencies among activities, and total cost estimation of risks.

In the case of unique, complex and lengthy projects, the schedule structure is often changeable, and the sequence of planned activities is not absolutely definite. While in Operations Research (OR) there have been efforts to build dynamic models that deal with variable schedules, a static sequence of activities, adopted from the BBT official documentation [Alf09], is being considered throughout the remaining discourse. Given a fixed schedule, possible adverse events related to the various tasks can be assessed and placed on the project mapping during the risk analysis process. Nevertheless, the existence of parallel activity lines, as well as partial exchangeability of tasks within these lines, offer a significant degree of flexibility, wherefore minor modifications and rearrangements still remain possible.

The aforesaid procedures can pave the way to more reliable economic estimations. The total cost of a project TC can be schematically[4] expressed as the sum of three principal terms, namely the base cost B, the risk part R, and a term F encompassing financial issues (inflation, exchange rates, present value adjustments, etc.) [PS06]:

$$TC = B + R + F \tag{2.2}$$

The present study is devoted to the calculation of the risk term R, which includes all individual risks and opportunities, R_j. The base cost B expresses the cost that will occur "if all goes as expected" [RB04]. Obviously, this definition cannot account for planned activities with variable cost; assuming a fixed value for such events usually leads to rather optimistic estimates. As the analyst attempts to account for variability, the borderline between base and risk cost becomes blurred, and the issue of a structured risk policy begins to flesh out. With respect to that, a classification

[3]Risk analysis textbooks and a vast number of relevant research papers refer to the risk management steps, providing different schemes. For instance, one may refer to [AM97], [CT01], [ETKV04], [KI07], [OO10], [Sch07], [SMJ06]. Nevertheless, an agreement on a certain procedure is rather a matter of standards.

[4]The plain arithmetical summation covers only the simplest case of this schematic aggregation. In practice, the three terms can have a more complex relation in the calculation.

can prove useful; risks can be divided into the three categories [SMJ06, p. 4]:

- *Known* risks: minor or significant known cost variations.
- *Known unknowns*: foreseeable events, whose occurrence can be assessed in terms of likelihood and impact.
- *Unknown unknowns*: unforeseeable events, usually deemed as "force majeure".

As the project advances from the conceptual and feasibility phases to those of design and construction, uncertainty typically diminishes [Jaa01], hence the total cost estimate approaches the real figure. This can be justified by the fact that an increasing number of planned activities have already been completed, rendering the associated risks irrelevant. Moreover, accumulation of knowledge and experience enables more efficient management of uncertainty. Therefore, the estimation accuracy and precision should be always considered with reference to the desired phase. This effect is often referred to and depicted as a *cost trumpet* [PSP07], [Ker09, p. 671]:

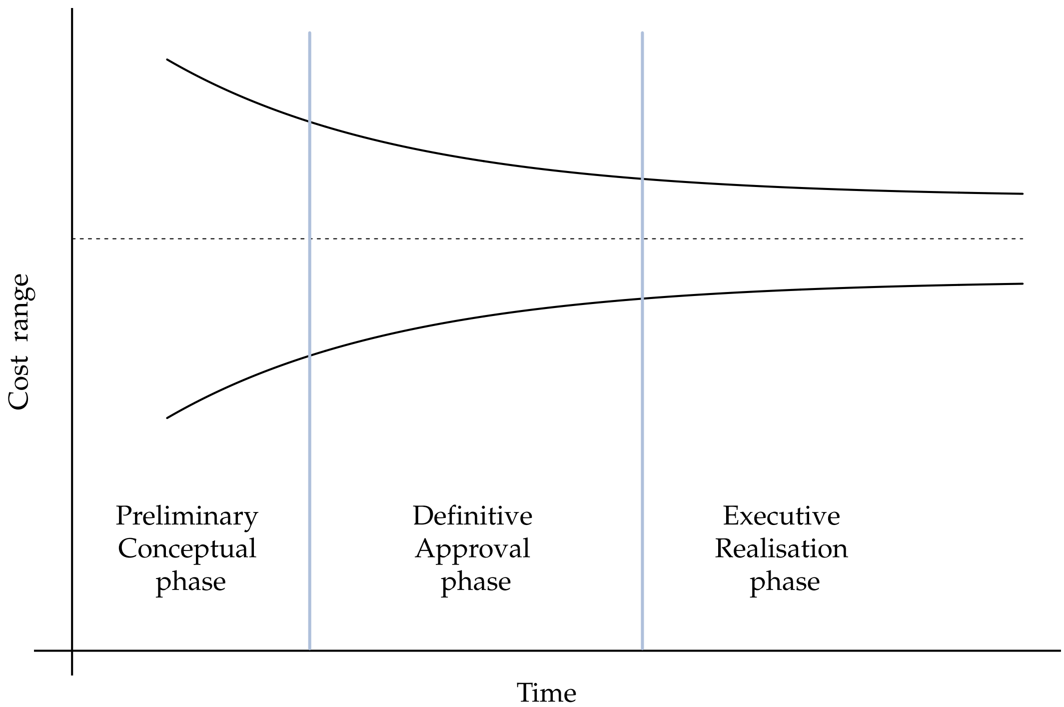

Figure 2.1: Cost "trumpet" graph: cost uncertainty is diminishing with time, as the project progresses. The description of phases is indicative.

As mentioned above, risk quantification bears, to some degree, upon the operator's ability to cope with the consequences of risks, or to influence the underlying mechanism. Zhang [Zha07] asserts that a consequence–based treatment of risk may tend to neglect vulnerability (the capacity of a system to respond to an event), for several reasons. For instance, direct causal relations between events and consequences are unsuitably presumed; moreover, human or organisational reactions towards risks are assumed to be systematically rational. Whereas the study of vulnerability may contribute to understanding theoretical aspects of the risk analysis process, and can prove beneficial in cases where the life–cycle of the project is considered in a broader socio–economic framework, the usefulness of the matter is rather limited when estimating the cost of a well–defined set of activities. Therefore, in the following, vulnerability is assumed to be inherently present in the given expert's assessments.

2.2 Types of risk

The types of risks encountered in a project, strongly depend on the application. Factors related to the social, cultural, economic, technological and political environment within which the project is conceived and constructed, are often referred to as *global* risk factors [DBH07]. Other factors specifically pertain to underground construction works; e.g. tunnel collapses [ETKV04], insufficient material deposits, etc. Also, an engineering mega–project is inevitably exposed, to some extent, to possible catastrophic risks.

In the case of large infrastructure works, one can observe at least [RB04], [Sch07], [AM97], [PS06], [Fin07]:

- Risk related to construction, stemming from poor engineering or from unforeseen natural hazards.
- Legal and political risks (legislation, regulations, guidelines, approval).
- Managerial and logistics risks.
- Environmental risks.
- Risk related to human safety.

With respect to the result of risks on the project's objectives, one can observe:

- Risks that seriously threaten the project's final completion (cost overruns, unexpected delays, market turbulence, etc.)
- Risk of not meeting the required standards (design, operation, maintainability, quality).
- Risk that the project may become technologically or operationally obsolete within its defined life–cycle.
- Risks related to anticipated revenue operations and expected utility.

2.3 Uncertainty

Randomness and uncertainty are inseparable from nature even in the simplest perceptions. The concept of handling ambiguous quantities and processes has been diffused in the past decades in many disciplines. Towards this approach, uncertainty models are nowadays being widely used, aiming to incorporate available information, as well as lack thereof, regarding variables and inter–variable associations. Within this framework, model inputs are being assumed stochastic in nature, described by probability distributions, fuzzy sets, intervals, or other mathematical tools.

Uncertainty is often regarded by non–technical practitioners and stakeholders as a synonym to risk, as a threat to the project's objectives, or even as lack of quality. On the contrary, when included in a risk analysis process, uncertainty can help the operator, not only to relieve from possible impact, but also to increase the level of control and awareness [ZZW07]. Moreover, the shift to an uncertainty–based paradigm, such as the Probabilistic Cost Analysis (PCA), has proven to be a step towards more reliable estimations. Uncertainty is present today in various forms, not only in academic research, but also in commercial software decision tools on the market[5]. Indeed, uncertainty–aware and risk–informed decision making is nowadays increasingly considered as an indispensable factor in important infrastructure planning.

Uncertainty is being typically classified into *aleatory* and *epistemic* [Ang10], [DKD09]. Aleatory uncertainty describes the variability in the observed data, and is not reducible since it reflects the randomness of the natural phenomenon of interest. Epistemic uncertainty arises from lack of information, or from inability to establish accurate predictive models; it can be controlled, albeit not always measured in absolute terms. In fact, this type of uncertainty can be reduced through an

[5] Any specific reference to such products in the present study has been suppressed.

advance in relevant knowledge, e.g. through scientific and technical development, or accumulation of experience. Allowing randomness of effects leads to stochastic outputs, whereas acknowledging information and model defects, places these outputs within a possible range. The concepts of aleatory and epistemic uncertainty will appear later in Chapter 3, as useful tools to construct probabilistic representations of cost items.

Several researchers have attempted further classifications, e.g. by categorising epistemic uncertainties as parameter, model, and completeness–type [VLDD$^+$12], [KIL$^+$10]. In any case, knowledge on the idiosyncrasy of various uncertainties may prove valuable, since it enables implementation of appropriate modelling principles [Win96], [Nik04]. Accordingly, irreducible variations are accepted as such, and integrated as model inputs, whereas gaps in knowledge impel the carrier to acquire more relevant data, or to engage more sophisticated predictive tools.

Apart from the type, a remark on the sources of uncertainty is also sensible. General sources are [BC00, p. 87], [CLP07]: subjective judgement, linguistic imprecision, statistical variation, inherent randomness, modelling defects, undetected causalities, lack of information, imperfect problem formulation, spatial fluctuations, measurement errors, etc. However, due to the diversity and multitude of uncertainty types, theoretical classifications alone possess little practical value. Since the present study is primarily concerned with the economic impact of risks which vastly relate to underground works, the generic sources of uncertainty should be restated on a pragmatic basis.

In underground projects, problems often result from inadequate geological data, inappropriate interpretation of conditions, or incompetence in dealing with arising issues [HP98]. When it comes to geomechanics, rock and soil are two materials containing significant uncertainties regarding their properties and failure mechanism. Soil samples are inavoidably disturbed when extracted, so the conditions applied in laboratory experiments may fairly differ from the corresponding conditions in situ; even worse, these conditions may not even be reproducible. Moreover, there are often large deviations within parameter values obtained in comparative studies [OF05]. Allowing parameter variability cannot necessarily account for these uncertainties[6], since local singularities also decisively affect the rock and soil conditions, as well as the corresponding engineering decisions.

The influence of uncertainty has proven to be even more intense in singular underground projects, such as tunnels [RB04]. Large tunnelling projects usually suffer

[6]The postulation that a probability distribution reflecting several diverse effects can be impractical, has given ground to possibilistic and fuzzy representations, mentioned in Section (3.10).

from lack of adequate information [DBH07], owing to the very nature of such laborious engineering works, as well as to often improper mode of addressing issues related to underlying uncertainties. Field experience with respect to engineering uncertainties has been reported in several publications. For instance, Ehrbar et al. [EBNB10] outline some typical doubts regarding earthworks, related to the trustworthiness of geological prognoses, the influence of subsoil conditions on operation advancement rates, and time delays due to major (hydro)geological events. Hoek and Palmieri [HP98] manifest that, in cases of long tunnels through mountainous terrain, it is impractical to obtain sufficiently reliable data from boreholes and exploration adits to investigate all the rock units along the route. Estimating geotechnical parameters through interpolation can be problematic. Therefore, the operator needs to achieve balance between incomplete knowledge of underground conditions and exploration or experimental costs.

The importance of distinguishing between risk and uncertainty has been recently highlighted in several studies [WC03], [PGW08], [Jaa01]. The shift of focus from risk as threat to uncertainty is not confined to a merely theoretical discussion, but aims to contribute to the development of more efficient processes. In fact, an uncertainty–based risk management can help to:

- Encourage quantitative over qualitative considerations.
- Draw a clearer distinction between causality and covariation.
- Allow the use of more generic and widely tested computational techniques, instead of case–by–case treatment.
- Broaden the spectrum of considered events[7].
- Enhance knowledge management.
- Acknowledge the need for opportunity management.
- Remove any possible "preconceptions about what is desirable or undesirable" [WC03].
- Follow the dynamic nature of the project's variables, since the degree of uncertainty depends on the time and phase.

[7]For instance, while highly uncertain parameters may have significant impact on the project's performance, they are being often left out as they don't constitute events, in the traditional sense [ZA97].

- Evaluate combined effects and factors by directly tracing their impact on the final quantities.

2.4 Levels of information

Information is closely related to uncertainty. As mentioned in Section (1.2), information is generally regarded as any element of knowledge, able to modify a decision. In tunnelling projects, useful data can be acquired from existent sources of geological information: surface mapping, subsurface investigations and analogous experience [HP98]. However, due to the limited availability of "hard" data, the analyst often has to resort to expert opinions [CW99], discussed in the forthcoming section[8]. At least the following three levels of information, with regard to risk assessment, can be identified:

1. Ignorance, where no essential information on the likelihood of occurrence, or the impact of a considered risk is at hand. This applies to *unknown unknowns* (unforeseen events), as well as to extreme rare events.

2. Qualitative information, originating from subjective judgement. It mostly resides in individual perceptions and is expressed in linguistic terms [CT01].

3. Quantitative information, usually in the form of probabilities, correlation coefficients, and empirical factors, obtained from historical data or normatively processed data acquired from the actual project.

The requirement for quantitative methodologies is inextricably linked to the concept of *measurability*. In Economics, quantities not easily measured, such as flexibility, productivity, or value of information, are called *intangibles*. Omitting such intangibles from risk analysis, or assigning numerical values of equivalent weight to every tangible and intangible system component, represent the two opposite attitudes against measurability. A huge amount of available and potentially useful information is typically ignored in traditional risk management, partly due to inability or reluctance to measure intangibles.

The increasing demand for handling information has recently given ground to the development of relevant methodologies. The term *knowledge management* (KM)

[8] Knowledge *elicitation*, the process of interacting with an expert, is only a subset of knowledge *acquisition*, the broader activity of gathering and managing information relevant to the project [WF93].

is used to describe the formalised process of acquiring, evaluating, documenting and transferring knowledge. A systematic KM can contribute to risk management by documenting and transferring experience from past projects [KGM08]. In fact, the need to improve organisational capabilities and reduce uncertainty involved in the planned activities is better served within a KM framework [WC03]. This can be a challenging task, since knowledge in a large–scale project is typically spread over many individuals, in multiple organisations and levels [PH02]. In this respect, a cost engineer is not only a specialised accountant, but also a "knowledge engineer".

2.5 Expert judgement

An *expert* is an individual who possesses professional knowledge of particular matters and processes, and substantial experience in his domain at a considerable level [BFM+06]. Experts are employed to collect, evaluate, interpret and communicate information, predict the system's behaviour, and assess uncertainties [ZA97]. An expert judgement elicitation process usually involves at least a substantive expert, mainly interested in the physical effects, and a normative expert focusing on the quantification part.

Over the last years, subjective judgement has been recognised as an invaluable source of information in the presence of uncertainty, and is nowadays considered as being complementary to, or even substitute for conventional approaches. Expert knowledge is a welcome alternative source of information on matters for which uncertainty elaboration is desired, but on which direct measurement is infeasible or impractical [GF00]. The acquired information can provide further insight to engineering and managerial decisions, scenario development, resource identification, experimental design, model selection, probability encoding, and numerical assessment. Complex and multidisciplinary projects can largely benefit from formal and proper elicitation, representation, and documentation of experts' conceptions and opinions [BM04].

For all its broadly acknowledged virtues, expert judgement in current practice is mostly informal, implicit and undocumented [ZA97]. There are many problems associated with a subjective opinion setup, the most important being the translation of judgements into probabilistic information [Rei02] or, more generally, into explicit representations [BM04]. Moreover, risk perception is influenced by people's own beliefs, awareness, attitude, judgements, and feelings [AM97].

A topic that has received great attention in the field of risk management is the existence of bias in expert opinions. In theory, professionals' opinions should be unbiased, realistic and practical; however bias, ambition and optimistic attitude interfere in human perceptions, and expert judgement is no exception. The methodological treatment of the matter by several researchers [WF93], [OBD+06], [Coo91], [Gig04] has made possible to identify and reduce bias through controlled processes. Several types of bias in expert judgements have been studied; for instance, *hindsight* bias is the inability to correctly recall one's own prior assessment, after new information is observed ("they knew it all along" syndrome) [BW09]. *Overconfidence* bias, owing to a series of factors (optimism, "impress your boss" effect, hidden incentives, market forces), and bias due to rationalising reduction of supposedly "inflated" estimates may also occur [KAE04].

Variability is observed, not only within experts, but also among experts. In general, there are several consistency problems involved in experts' assessments, owing to different backgrounds, assumptions and interpretations [ZA97]; these issues appear as [OV06]:

- Incoherence within experts, when the estimates by a single expert are problematic.
- Inconsistency among experts, when contradictory assessments are assigned from different experts to the same risk.
- Incoherence among experts, when incompatible probabilities for logically related risks are assigned by different individuals.

When multiple, often conflicting, experts and beliefs are involved, the fundamental problem of concluding to a final aggregate assessment naturally arises. Clemen and Winkler [CW99] manifest that, "if experts never disagreed, there would be no point in consulting more than one". It has been reported [Ada06] that weighted aggregates of estimates are more accurate than the individual subjective estimates that comprise the aggregates. This position is reasonable, only in analogy with the fact that a weighted mean is better informative than the individual observations, as long as the reliability of the gathered information is adequately controlled.[9]

Several behavioural and mathematical approaches have been proposed by scholars (e.g. [CW93]), for aggregating multiple expert opinions. Osherson and Vardi [OV06] used the term *aggregation principle* to express the method used to convert multiple expert opinions into ultimate judgements, and to address relevant logical challenges. Problems associated with these techniques lie in difficulties to account

[9]However, seeking the truth by combining or bisecting false statements, does not guarantee a successful outcome.

for possible dependencies among experts and to propagate individual uncertainties in the composite assessment [ZA97].

A popular approach, dating back to 1950's but still widely in use, is the *Delphi* method [LT75], [Ada06], [PC96]. According to this method, experts involved in the assessment of a certain risk are brought together in thematic meetings, where they are further informed on the risk in question. Then, they develop independent estimates about the issue and report their opinions to the central group. Based on feedback and discussion, the reinformed experts confirm or revise their estimates until consensus is reached. The main assumptions in all Delphi technique variations are that (1) the individuals possess a considerable level of expertise on the matter, and (2) they independently develop their judgements. The described methodology was followed in the risk assessment of the case study.

2.6 State of the art

Current size and complexity of engineering undertakings often exceed the traditional management techniques. In particular, tunnelling projects often suffer from cost overruns and schedule slips [FHB02], usually attributed to unforeseen events. Such discrepancies may even occur in projects with proper design and construction achievements [RB04]. These problems call for an effective treatment of risk, uncertainty and variability in important engineering works. Consequently, integral risk management practices are often nowadays employed; these processes can be notably refined by the use of methodical techniques, throughout the project development. Systematic risk management enables to identify potential problems and allow timely implementation of appropriate mitigation measures [ETKV04]. To that end, risk classification techniques are used, e.g. the hierarchical risk breakdown structure (HRBS), or more generic work breakdown structure (WBS) [Cha01], [Gar99, p. 254], [ZPA08].

However, formalised risk management is still absent in many construction organisations [CT01]. In particular, risk assessment processes have not been widely applied to cost estimating for large engineering projects [RB04], where focus mainly remains on preventing hazardous events. Since singular and latent risks inevitably appear in tunnelling works, they must be indirectly included in cost estimations; yet current approaches tend to overlook this fact, and to design upon idealised assumptions.

Akintoye and MacLeod [AM97] provide a list of reasons for which contractors and managers do not use risk analysis techniques. According to their survey, these reasons include, inter alia, lack of familiarity and expertise, constraints on time and information, and doubt regarding the applicability and benefits. Furthermore, different involved parts exhibit likewise different priorities concerning the importance of risk management. Difficulty in obtaining input estimates and probability assessments is also mentioned [BS99]. Similar findings are reported in other analogous works [ALA04], [LS04].

Despite the fact that Risk Analysis became a popular topic in academic research in the 1990's [TYL11], its value is still not fully appreciated in the realm of Civil Engineering [FS03]. The construction industry has been slow to realise the potential benefits [UT99], whereas organisations operating large construction projects usually adopt a limited scope of risk management, by merely prescribing ab initio a fixed, stiff response protocol [GKK10]. In fact, risk assessment is often viewed as an distinct and static process, rather than as a component of all decisions, permeating the project's life–cycle [Jaa01]. Next to that, restricted focus on the management of uncertainty is induced by the risk–oriented approach [WC03]. With regard to dependencies among risks, there is a difficulty to detect, understand and measure the relevant concepts [BS99].

2.7 Challenges in large construction projects

A mega–project comprises a number of sub–projects which serve the same strategic goals in a specific sector or geographical region, although they may differ in objectives or activities [ZPA08]. The realisation of a project of such magnitude and importance appears always as a challenging venture. There are several factors regarding the viability of a large construction project, with the following three being particularly critical [Fin07], [CFR07]:

Technical feasibility

Unique and complex infrastructure undertakings require detailed studies in order to ensure that the planned processes and the designed facilities are possible in the given technological, physical, economic, spatial, and temporal constraints. These studies must cover a multitude of engineering works, from design and construction to operation and maintenance. In particular, large tunnelling projects typically face significant—often atypical—technical and engineering challenges [ZZW07], and require interdisciplinary expertise and scientific innovation.

Economic viability

Economic viability is ensured when the generated profit and benefit during the expected life of the project is anticipated to be greater than the total expense. Such a comparison is neither restricted to technical provisions only, nor is merely a monetary account of known quantities. It must include factors that are often intangible or ambiguous, such as human safety, environmental risk, or quality of life [KP09]. Even in case a project's viability has been technically verified on a cost–benefit basis, there are inevitable limitations regarding its funding and the amount of debt it can raise.

Ecological sustainability

Infrastructure projects inevitably disturb the flora and fauna of the nearby location and modify environmental aspects to some extent. Therefore, any relevant adverse (side–)effects, generated by the project, need to be studied and managed in advance. This requirement broadens the scope of the planned works, which can often exceed the technical ability of the operator.

As mentioned in the preceding section, tunnelling projects frequently undergo cost and schedule overruns. In fact, budget excesses in complex infrastructure projects (even up to 100%) are quite frequent [PS06], [NS08, p. 290]. As seen in Equation (2.2), the total cost is not calculated simply as the sum of expenses that are directly linked to the project (i.e. materials, labour, equipment, etc.). Such accounts are inevitably based on several assumptions, and are restricted by copious physical or operational limitations. Disparities may occur due to a variety of technical, managerial, financial, social and political factors [HP98]; in particular, reasons for cost overruns in large–scale and long–term construction projects, are [RB04], [KAE04], [NS08, pp. 291–295], [PS06], [Ada06], [EBNB10]:

- **Variability**: The total cost is subject to a large number of variables, assumptions, conditions and requirements; these variables are not directly controllable or absolutely quantifiable. The input variability may, in turn, generate a broad range of probable cost. Moreover, accumulated—often neglected—minor modifications, can have a significant impact on total cost. Deterministic attitude and disregarded or optimistically assessed uncertainties, intensify the problem, whereas deficient treatment of cost variations (e.g. use of improper probability distributions) has also been reported. Finally, changes in objectives, new unplanned requirements and managerial transitions can yield unexpected costs.

- **Complexity**: Several factors cannot be initially handled in terms of cost and time; even worse, cost and time are obviously not independent[10]. Likewise, interrelations among cost elements may have a significant impact. Complexity can lead to poor project definition, posing severe threats to planned workflows.

- **Applicability**: Pressure to produce doable estimates, ineffective procurement processes and inadequate allocation of budgets may encourage unrealistic low bids. Weak management or strict policies can lead to infeasible demands in terms of time and cost. In the engineering field, one–off technical works may demand non–standard approaches and significant effort in Research and Development (R&D).

- **Ignorance**: Large–scale tunnelling infrastructure projects constitute prototypes, often suffering from lack of suitable comparable data. The majority of construction risks are subjective, in the sense that their assessment is mostly based on beliefs about the risks. Incorrect cost estimates and major delays may appear due to insufficiently known geological and hydrogeological conditions. Use of inadequate information elicitation methods is also frequently observed.

- **Opportunity**: The estimation of cost also includes a number of opportunities, which are often intangible utilities, dubiously projected in future times. The evaluation of opportunities is performed within a broad socio–economic environment, usually not possible to simulate and control in a normative manner.

- **Multi–objectivity**: A large project typically has multiple aims, purposes and objectives, which can generate conflicts and incompatibilities.

[10]This problem has given ground to time–cost tradeoff (TCT) methods [Yan11].

Chapter 3

Evaluation of individual cost elements

3.1 Risks as individual cost elements

The cost estimation process in a large–scale and long–term project usually follows a combination of analytic and synthetic reasoning. Firstly, the project plan is decomposed and analysed (disaggregated), e.g. through an activity breakdown procedure [Hil02]. Then, the individual cost items are identified, evaluated and documented. Finally, the total cost is computed as the synthesis of the assessed elementary costs. The first step is a project–specific task, typically performed during the planning phase. The second step is the subject of the present section, while the third is theoretically examined in Chapter 4 and further practically investigated in Chapters 5 and 6.

In the present context, individual cost elements fall within one of the following categories:

- Costs with figure deterministically known or, at least, believed to be so. The sum of these items yields the base cost B.
- Planned tasks or activities with uncertain cost, hereafter referred to as *variable* costs.
- Events with assessed likelihood of occurrence and economic impact (known unknowns), in the following called for shortness *singular* costs.
- Cost items of any type, not present in the actual assessment (unknown unknowns), in the present work referred to as *latent* (or *residual* [GKK10]) costs.

Since this study is primarily concerned with the evaluation of risks, only those activities identified as (or associated to) risks are considered. According to the definition in Section (2.1), "*risks (and opportunities) are economic items, affecting the project budget, that cannot be deterministically known, since they are associated with events, decisions, processes and interrelations under conditions of randomness, vagueness and ambiguity*". This definition applies to variable, singular and latent costs, which can be treated as random variables or, more general, as "uncertain quantities" [CG04].

Several approaches can be employed for the description of individual risks. In most of these methods, the common objective is that each uncertain input be modelled by a probability density function (PDF); this probabilistic cost analysis (PCA) approach is also adopted in the present study. The parameters of PDF's (mean, variance, mode, shape, etc.) can be established upon theoretical assumptions, historical data, expert opinions, or external evaluation of these opinions [FPR09]. Besides that, the characteristics of the PDF's family are dictated by (often debatable) requirements, discussed in detail in the remainder chapter.

With respect to the aforesaid requirements, some general principles need firstly to be agreed upon. In order to implement a PCA approach for describing risks, a series of modelling decisions should be justified. The selected model should [IN09], [MOD12], [Ave11, p. 38]:

- Rely on sufficient information, while integrating the most possible thereof, for constructing probability densities.
- Allow for treating interdependence among risks, including detection, quantification and propagation.
- Contain sensitivity investigations to account for model, human and numerical uncertainties.
- Exhibit flexibility to be modified, adapted or extended in the light of new evidence.
- Be robust, in the sense that small variations of the input data or slight modification of assumptions and conditions do not produce large deviations in the output.
- Yield clear, interpretable results through a transparent calculus with reasonable computational means and burden.

- Be able to undergo conceptual, experimental and operational validation.
- Constitute a basis for drawing sane conclusions and making effective decisions.

An important concept used in the evaluation process in the BBT risk analysis, is the risk's *evaluation basis* (or *baseline* cost). It can be defined as the part of the project budget threatened by the individual risk of concern. Once this figure is determined, the related risks can be assessed as fractions of the evaluation basis. A risk's evaluation basis can be obtained only when a preliminary form of the project has been designed, and prior cost calculations, including scenario evaluation, has taken place.

In the remainder section, the four most suitable and commonly used techniques to obtain information for the construction of univariate cost distributions, are exposed. In Section (3.2), the uncertainty pertaining to the impact bounds is discussed. Section (3.3) is dedicated to the confidence in assessment—an often disregarded type of uncertainty—and relevant ideas, developed during the present research. The typically proposed and employed probability distributions in analogous applications are outlined in Section (3.4). In Section (3.5), the problem of selecting appropriate univariate densities is analysed in the light of an extensive literature survey. The beta distribution, build upon a novel concept of confidence, is studied in Section (3.6). Other modelling approaches are briefly reviewed in the closing Section (3.10).

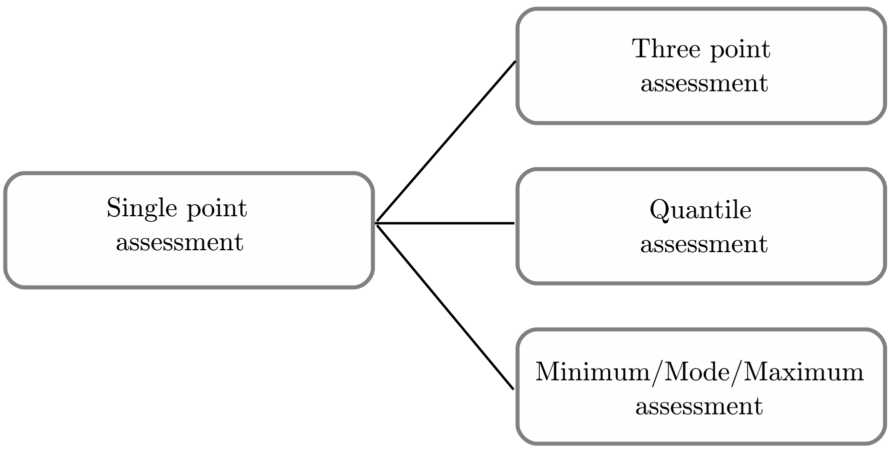

Figure 3.1: Outline of common techniques for individual risk assessment: single point assessment and three possible extensions.

Single point assessment

In some cases, the risk impact can be estimated only as a single figure, either as a percentage of the risk's evaluation basis, or as an absolute economic value. This is apparently the simplest way of quantifying an identified risk. However, a single value cannot realistically represent a variable or singular cost, since it provides no indication of uncertainty [CG04]. Whereas single point estimates for cost elements are inevitably approximate, they can be further expanded in order to account for variability. For such cases, e.g. a gaussian or a symmetrical beta density can be used in place of the point estimate (or a skewed one if there is enough evidence to assume a departure from symmetry). This technique, sometimes referred to as *contingency allowance*, is commonly encountered in current practice [Elk00].

The point estimate can be set as the mean value of the desired density, while the coefficient of variation CoV may be specified ad libitum, according to the level of confidence in assessment[1]. At this point it is worth mentioning that, whereas replacing a single value estimate with a probability density by assigning a CoV value may seem as an arbitrary decision, retaining only a fixed value is equivalent to setting $CoV = 0$. The latter is a specific choice, consciously unrealistic, concerning the risk's variability.

Eventually, a sensitivity analysis on the CoV value can be performed, in order to investigate the influence of this uncertainty metric on the final results. In case this influence is relatively significant, the point estimation is probably inappropriate, and more careful assessment with respect to the risk under study is required. A method for constructing probability densities for these cases is proposed and discussed in Section (3.6).

Three point assessment

A single point assessment aims to determine the most likely value, which corresponds to the most probable instance of a risk. When a number of possible outcomes of the assessed event can be identified, it is natural to attempt an ordering based on their expected impact. This approach is as a natural extension of the single point assessment; it relies on the evaluation of more scenarios regarding the risk, instead of only the average one.

In a simple form of this technique, the analyst is tempted to define three characteristic values for the risk impact I (minimum, medium and maximum/downside) and to ascribe occurrence probabilities. Since the sum of the probabilities is unity

[1] This concept is discussed in detail in Section (3.3).

(as condition (3.1) requires), five values in total have to be assessed; finally, a discrete distribution is constructed. If the ordered impact values are denoted by $I_j, j = 1, 2, 3$ and the probabilities by $P_j, j = 1, 2, 3$, then:

$$P_j = P(I = I_j), j = 1, 2, 3$$

$$P_1 + P_2 + P_3 = 1 \tag{3.1}$$

An immediate question is whether the assessed discrete impact distribution can be replaced by a continuous one. In cases when the nature of the risk implies a continuous variable, such a replacement is a reasonable choice. In other cases, the discrete distribution may appear to be more representative; this idea has also been supported in [Yan05]. The desired density can be constructed using the statistical moments of the assessed discrete density (expected value, variance, or higher order) [DKLM07]:

$$\mu = \mathrm{E}[I] = \sum I \cdot P$$

$$\mathrm{Var}[I] = \mathrm{E}\left[(I - \mu)^2\right]$$

Standard quantile assessment

Several variations of this method are being widely used in risk assessment [LWW08], [OO10], [ETKV04]. Here the expert is not asked to explicitly specify probabilities, but to assess standard predefined quantiles of the impact distribution. The number of estimated values, usually three to five, depends on the level of available information and the importance of the particular assessment. In a simple form of the technique, the expert is asked to answer the following questions:

Q_1 : *"What is the impact value that is unlikely to not be exceeded?"*

Q_2 : *"What is the median risk impact value?"*

Q_3 : *"What is the impact value that is unlikely to be exceeded?"*

Question Q_2 has received much attention in uncertainty analysis. It is well–known in elementary statistics that the median, the most likely value (mode) and the mean (expected/average) value are different in general. However, the distinction in the context of expert judgement is not always evident. For instance, when an expert is asked to provide a modal estimate, confusion between the mode (not a quantile) and the median (a quantile) may appear [LS95]. Research has shown [AAG98] that the median is a quantity which people can assess reasonably well. Moreover, the median is less sensitive to the tails of the distributions compared to the expected value [VLDD+12]. Still, there is no clear indication in the literature whether subjective assessments of the median are more reliable compared to the mode. In the above formulation, the median is used mainly so that consistency among the questions be preserved.

Another problem in this context lies in the linguistic vagueness of the term "unlikely". Experts asked to assess quantiles need not all have consistent quantitative perceptions. Even if the term is used to express a certain likelihood (e.g. $P = 10\%$), the problem does not vanish; not all experts possess the same mechanism of transferring degrees of belief into probabilities. This diversity appears to be more intense for small values of P [OBD+06, p. 133], [dRDT08]. Therefore, the use of 10% has been postulated as probably safer, compared to lower quantiles [Joh97]. In this case, the answers to the three above questions are:

A_1 : The 10% quantile I_{10} of the distribution: $P(I \leq I_{10}) = 0.10$.

A_2 : The 50% quantile I_{50} (medium) of the distribution: $P(I \leq I_{50}) = 0.50$.

A_3 : The 90% quantile I_{90} of the distribution: $P(I \leq I_{90}) = 0.90$.

If the questions Q_1 and Q_3 are formulated with the wording "impossible" instead of "unlikely", then two more quantiles I_{100} and I_0 can be obtained. However, this is not always practicable in risk assessment, due to the vagueness of bounds, discussed in the forthcoming section. Anyway, these two new questions are:

Q'_1 : *"What is the risk impact value that is impossible to not be exceeded?"*

Q'_3 : *"What is the risk impact value that is impossible to be exceeded?"*

A simplified version of the above method was chosen for the analysis in the assumed case study. It is introduced in the remainder section, and further explained in the course of the present chapter.

Minimum – mode – maximum assessment

In cost and time engineering applications presented in the literature, the most common technique is the assessment of the following three values:

- The lower bound L (minimum).
- The most likely value M (mode).
- The upper bound U (maximum).

The assessment of these three values can lead to the construction of a probability density, depending on a number of additional assumptions dictated by the targeted application. The described technique is preferable compared to the three–point assessment, because it is free of probabilities and all accompanying problems. In most relevant studies, the most likely value (usually referred to as "mode") is used in place of other central values (median and average). In the present work, it was selected since it represents the quantification of the most likely scenario with regard to the risk of concern. This scenario is usually the basis of any further individual risk assessment. In the following section, the problems pertaining to the two bounds L and U are discussed.

3.2 The vagueness of bounds

A common practice in risk quantification, as mentioned in the earlier section, suggests that, for individual cost items, a triad of quantities consisting of an optimistic, the most likely and a pessimistic value for the anticipated cost impact can be obtained through an expert elicitation process. These values, denoted as L, M and U, can be derived as the outcome of evaluating an extremely favourable, the most expected and an extremely unfavourable scenario, respectively. However, there is no consensus in the literature on what the assessments L and U should correspond to [LS95], [Rei00]. Depending on the context of the problem, the ability of the experts, or the analyst's standpoint, they may represent either absolute endpoints or standard percentiles.

The minimum and maximum values are often characterised as being vague. For instance, in a reported a posteriori analysis of assessments [KAE04], 20% rather than 2% of the actual values were found to fall outside the 0.01 to 0.99 quantiles interval. In fact, most analogous studies indicate that the assessed bounds are usually rather tight and biased towards confidence. A typical explanation bears upon the appearance of anchoring effect, i.e. the tendency to assess bounds close to the initially specified mode [OBD$^+$06].

When the bounds cannot be assessed, or are deemed optimistically biased, the analyst is often tempted to empirically deduce them from existent information. A simple technique suggests the expansion of the PDF's initially assessed range $U_0 - L_0$ to both directions by a percentage specified by a constant e:

$$U = U_0 + e \cdot (U_0 - L_0)$$

$$L = L_0 - e \cdot (U_0 - L_0)$$

However, this account may result to negative values of L. When the uncertain quantity of concern is a risk (cost, time, or other consequence), it might be sensible to leave L unchanged and modify only the value of U.

As an alternative approach, the two missing bounds can be estimated by:

$$U = f_1 \cdot U_0$$

$$L = \frac{L_0}{f_2}$$

with $f_1, f_2 \geq 1$. The values of f_1, f_2 can be either discussed with the experts in a case–to–case basis, or assumed with a fixed value. It is worth noting that the second technique shifts the density to the right, therefore care should be taken when the modelled uncertainty has a positive influence on the objective quantity.

A similar approach can be taken when the initially assessed values L_0, M_0, U_0 represent a possibly undervalued risk. Then, the conservatively modified parameters can be introduced as:

$$L = f \cdot L_0$$

$$M = f \cdot M_0$$

$$U = f \cdot U_0$$

where $f \geq 1$ is an "inflation factor" that increments the assessed values by a certain percentage, reflecting the confidence, a concept further discussed in the forthcoming section. In the present study, the aformentioned techniques to deal with uncertain bounds are only theoretically outlined, but not implemented in the computation. Instead, the method introduced in the following sections is proposed.

3.3 Confidence in assessment

As highlighted in Chapter 2, uncertainties involved in the risk term R of the total cost can be of aleatory and epistemic nature; hence they can be reduced to a certain degree, yet not entirely eliminated. Identification of the origin and magnitude of input uncertainties is of great importance for the evaluation of any type of system response, e.g. the total cost [Win96], [NA03]. This is also stressed by the often postulated tendency of individuals to rarely assess uncertainty to be as large as it should [OBD+06]. The central objective of the present section is the clarification of two similar, yet not equivalent notions: the true uncertainty involved in a project realisation process, and the human uncertainty expressed in the experts' assessments.

True uncertainty results from possible adverse events, natural hazards, parameter fluctuations, physical randomness, planning modifications and other sources. It represents inherent attributes of the phenomena (aleatory), or effects of undisclosed information lying beyond the analyst's awareness (epistemic). In any case, true uncertainty can be known only to a certain degree, due to various constraints. Human uncertainty is any contingency consideration introduced in subjective judgements and expert opinions, in order to represent the perception of physical uncertainty.

While these uncertainties are often presumed identical for the sake of computational convenience, they may significantly differ, especially under conditions of high complexity and lack of information. On the one hand, true uncertainty is not a subset of human uncertainty; if it were, every element of true uncertainty should be also human. Such a claim would imply that the team of experts, engaged in the risk analysis process, functions as a powerful predictive mechanism, capable of identifying every possible adverse event and every variation affecting the process under study. Even given this ideal assumption, the translation of human perceptions into explicit probabilistic statements can be neither deemed lossless, nor straightforward [BM04]. On the other hand, human uncertainty is not a subset of true uncertainty; if it were, every element of human uncertainty should also be true. This would mean that every element of scepticism or apprehension in the experts' opinions is not unrealistic or abitrary, but genuinely corresponds to actual effects unsettling the project.

Let T and H denote the true and the human uncertainty, respectively. The set $T - H$ includes unforeseen events and true uncertainties not captured by the risk analysis process. The set $H - T$ could simply be labelled as "fear", since it contains assessed uncertainties that do not really exist. The assessment of uncertainty where

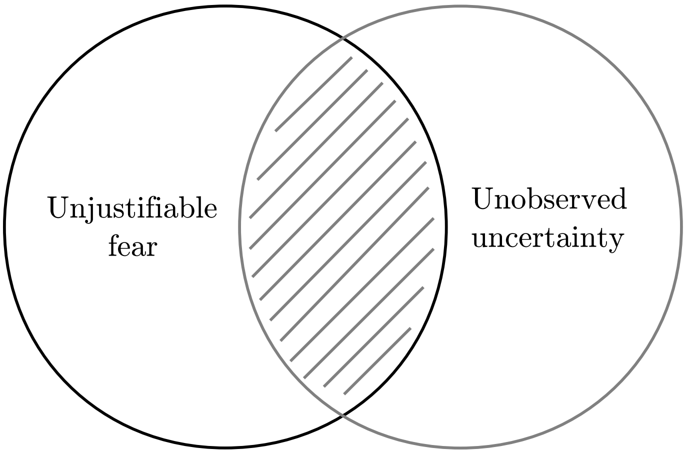

Figure 3.2: Relation between true uncertainty (left circle) and human uncertainty (right circle). The shaded intersection represents the uncertainty captured by expert judgements.

it really does not exist is closely related to the concept of "type I" errors, or "false positives". Likewise, the uncaptured uncertainty is related to the concept of "type II" errors or "false negatives". Thus, according to the previous arguments, the relation between these two uncertainties can be simply depicted in Figure (3.2).

Despite the difference in nature between these two distinct parts, they can be used interchangeably, provided they produce the same effect on the quantity of interest. In fact, this is the very rationale behind the use of expert judgement in the risk quantification process. In order to ensure this equivalence, a one–to–one mapping between the items of uncaptured uncertainty, which is always unknown, and unjustifiable fear must be established. Once this achieved, the known part of redundant human risk perception can be used to model risks not appearing in the risk identification process. Establishing the aforesaid correspondence requires prior knowledge available from similar previous projects or from already completed phases of the actual project. Thus, it is essential that the steps of the realisation process be a posteriori documented and analysed, and the obsolete assessments be reexamined in the light of new evidence.

Apparently, as the project progresses, these two parts are diminishing, as seen in Section (2.1) and depicted in Figure (2.1). After a certain project phase is completed, the resulted individual cost deviations from their expected values can be compared to the corresponding scale metrics introduced by the experts in the probabilistic cost analysis. Next to that, a qualitative comparison between the unwanted events that actually appeared, and the possible risks identified during planning can further assist the mapping between true and human uncertainty. Moreover, the reasons of any significant disparities can be analysed, providing valuable information for the forthcoming project steps. In simple wording, the described concept stands as a formalised "learning from own mistakes" attitude. A study on similar ideas alongside with an effective implementation on knowledge risk management can be found in [KGM08].

Every risk assessment and uncertain quantity estimate is accompanied by a belief on the imprecision of this estimate [WS97]. Obviously, the assessment of each risk is performed under different levels of confidence. The latter may reflect the inherent variability of the phenomenon under study, limitations of the predictive process, the quality of available information itself, or the expert's perception regarding the reliability of this information. Furthermore, confidence can be distorted by psychological factors, such as bias, as seen in Section (2.5). This confidence—regarded as a fuzzy conception describing attitude or psychological state—can be included in the uncertainty propagation process, in order to better achieve a balance between true and human uncertainties, and to acknowledge and express the limitations of assessment. Since probability is an expression of the confidence that an event will occur [BC00], the confidence in assessment can be also viewed as a second–order probability. Within this framework, both the substantive expert, who is mainly interested in the physical process, and the normative expert, who focuses on the quantification of the process, can contribute to the determination of the confidence level.

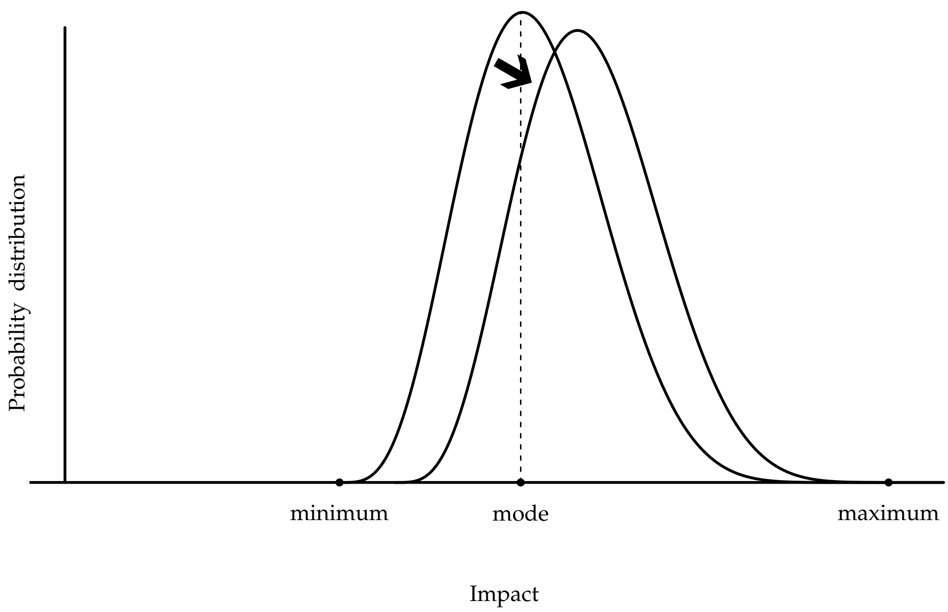

Figure 3.3: Inflated beta density with *L, M, U* multiplied by an "inflation factor".

The idea of quantifying the expert's confidence in assessment has been implemented in a few studies (e.g. [AAG98], [Hah08]), while various aspects and interpretations of the uncertainty in probabilistic assessments are highlighted in [OO04]. Reid [Rei09] manifests that it is necessary to address confidence issues within uncertainty modelling.

A frequent fallacy underlying the PCA approach is that all three assessed parameters (minimum, median, maximum) are equally significant and estimated with the same degree of trust. The importance of this assumption is implied in [vDK02]. With respect to that, two different ways are proposed in the following:

Confidence encoded in L, M, U

As discussed in Section (3.2), the lower L and upper U assessed values often prove unreliable, and have to be expanded. Thus, increased uncertainty can be introduced, depending on the confidence in assessment. In this case, one of the approaches mentioned in the aforesaid section can be used, resulted to a new uncertainty representation, such as the one in Figure (3.3).

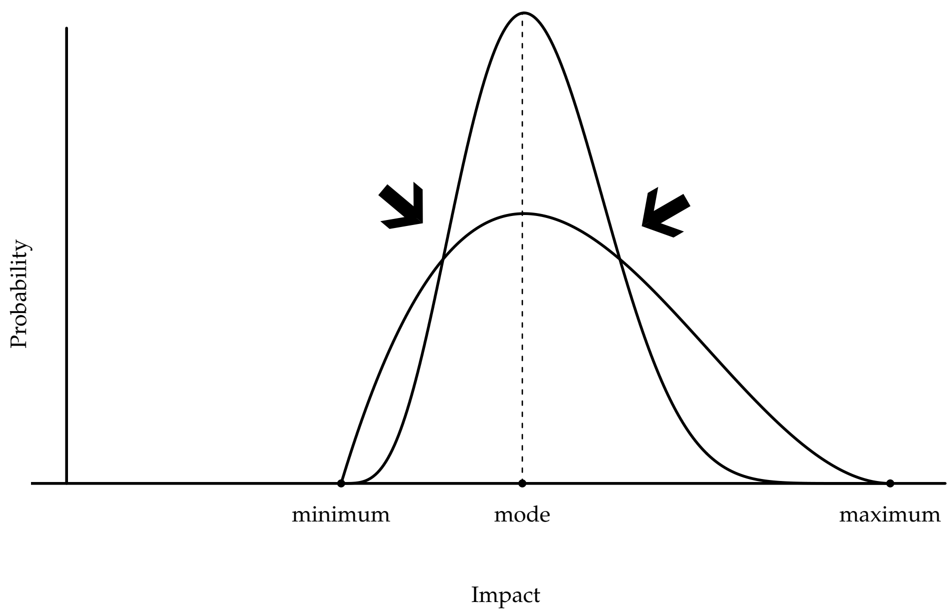

Figure 3.4: Beta density inflated by increased dispersion.

Confidence encoded in M

Case may be that the cost bounds L and U are determinate, but the uncertainty mostly lies in the value of M. In this case, the degree of concern can be encoded in the dispersion of the density. To that end, the coefficient of variation (CoV), which can be viewed as a measure for the lack of confidence in assessment, with regard to the most anticipated value M, can be specified. Introducing increased uncertainty results to "squeezed" distributions (Figure(3.4)). This technique is developed in Section (3.6).

3.4 Candidates of univariate distributions

As stated previously, the probabilistic cost analysis (PCA) route is adopted in the present study. According to this approach, variable and singular costs are represented by probability distributions. Latent costs cannot be literally modelled, since they correspond to unforeseeable events; however, their effect can be indirectly accounted for, e.g. by calculating "safer" estimates, in a probabilistic sense. The proposed procedure falls into the class of so–called "soft" probabilistic methods, since the PDF's represent possibilities, namely beliefs on one–off irreproducible events, rather than frequencies. However, the possibilistic factor is not restrictive within the mathematical formulation, but only necessary for the proper interpretation of the results.

Typically, two approaches appear in the literature for the construction of probability distributions through subjective opinion. The first suggests that the distribution can be build using assessed predefined quantiles, while the second proposes the assessment of the distribution's moments. Moment methods provide theoretical advantages; however, in practice, they rely on numerical information that cannot be elicited with accuracy, thus offering no real advantage [Rei02], [AAG98]. Moreover, when sample data are not available, the selection of an appropriate distribution family rests on expert opinion in a rather arbitrary manner [KLS$^+$06].

For the stochastic representation of individual risks as cost elements, the selected PDF density is generally desired to be smooth, unimodal, and able to achieve right–skewed, left–skewed and symmetric shapes. Since the densities play a possibilistic role, they are selected from the class of "mild" distributions—as opposed to some unusual examples found in the context of financial statistics, where hard data are available. In the literature, either closed–end (e.g. uniform, triangular, trapezoidal, beta) or open–end (e.g. normal, lognormal, weibull) probability distributions have been proposed[2] [FPR09], [WH00], [ETKV04], [Joh97], [KI07], [Yan07], [Yan11], [Pet12], generating a fertile debate. In the following, an inventory of these distribution families alongside with PCA considerations is presented, according to [Gar99], [JKB00a], [JKB00b]. The x–axis is labelled as "Impact", for the sake of scope consistency.

Uniform

A random variable X is said to have a uniform (or rectangular) distribution when the PDF is constant:

$$f_X(x) = \frac{1}{U-L}, L \leq x \leq U$$

The expected value and the variance of X are given by:

$$\mathrm{E}[X] = \frac{1}{2}(L+U)$$

$$\mathrm{Var}[X] = \frac{1}{12}(U-L)^2$$

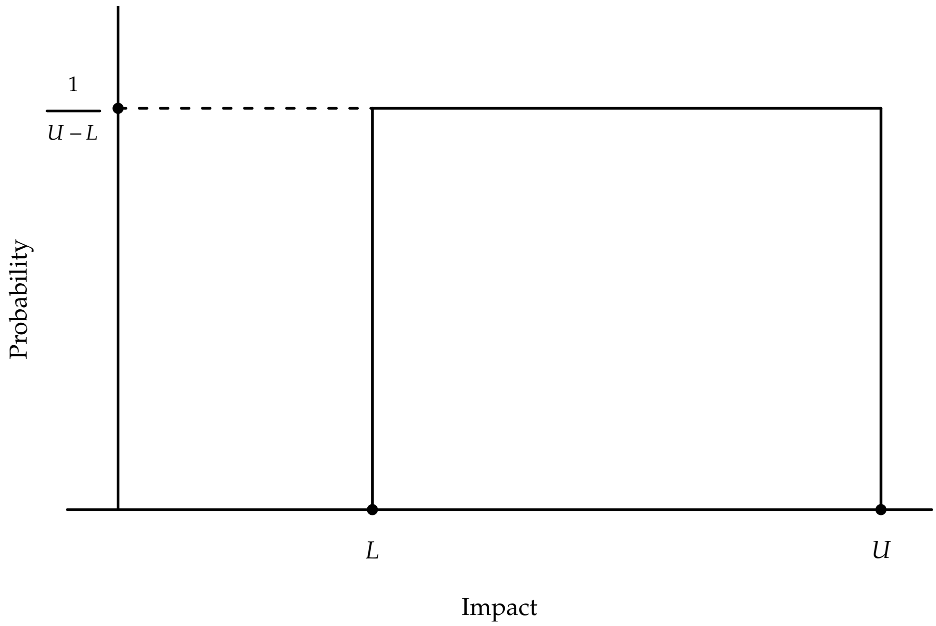

Figure 3.5: The uniform distribution.

The uniform distribution is not skewed and does not possess a mode. Therefore, it represents cases when a most likely value cannot be determined, but the bounds are

[2] Also, open–end distributions can be modelled and truncated afterwards [VLDD+12].

known [Ins08]. The coefficient of variation of the standardised (bounded in $[0,1]$) uniform density can be readily shown to be $CoV = \sqrt{3}/3 \approx 0.58$.

Triangular

The PDF of the standardised triangular (bounded in $[0,1]$) distribution with mode m in $(0,1)$ is given by:

$$f_Z(z) = \begin{cases} 2z/m & , 0 \leq z \leq m \\ 2(1-z)/(1-m) & , m \leq z \leq 1 \end{cases}$$

The general form of the triangular density, bounded in (L, U) after the linear transformation $X = L + Z(U-L)$ is given by

$$f_X(x) = \begin{cases} \frac{2(x-L)}{(U-L)(M-L)} & , L \leq x \leq M \\ \frac{2(U-x)}{(U-L)(U-M)} & , M \leq x \leq U \end{cases}$$

The expected value and the variance of X are given by:

$$\mathrm{E}[X] = \frac{1}{3}(L + M + U)$$

$$\mathrm{Var}[X] = \frac{1}{18}\left\{(M-L)(M-U) + (U-L)^2\right\}$$

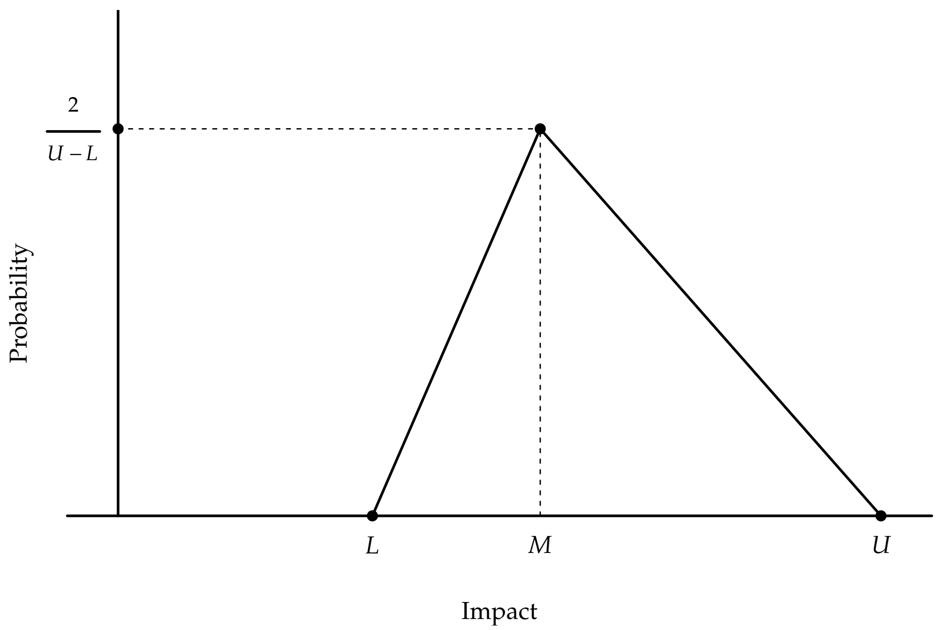

Figure 3.6: The triangular distribution.

The mean and the standard deviation of the triangular distribution are equally sensitive to all three parameters [Vos97]. The triangular is necessarily unimodal and can be symmetrical, positively or negatively skewed. It can be simply constructed from the elicited information (minimum, mode, maximum), and easily understood. However, it can yield excessively heavy tails when M is distant from U or L [KIL+10], [CGRW04]. Moreover, it includes no consideration regarding the uncertainty of the assessed parameters; these two shortcomings of the triangular can severely distort the desired output. The coefficient of variation of the standardised triangular density equals to $CoV = \sqrt{6}/6 \approx 0.41$.

Beta

The PDF of the beta distribution bounded in $[L, U])$ with mode M is given by:

$$f(x; \alpha, \beta, L, U) = \frac{1}{\mathrm{B}(\alpha, \beta)} \frac{(x-L)^{\alpha-1}(U-x)^{\beta-1}}{(U-L)^{\alpha+\beta-1}}, L \leq x \leq U$$

The beta family is given with more detail in Section (3.6).

Trapezoidal

The PDF of the trapezoidal distribution is given by:

$$f_X(x) = \begin{cases} \frac{2(x-L)}{(U+M_2-M_1-L)(M_1-L)} & , L \leq x \leq M_1 \\ \frac{2}{(U+M_2-M_1-L)} & , M_1 \leq x \leq M_2 \\ \frac{2(U-x)}{(U+M_2-M_1-L)(U-M_2)} & , M_2 \leq x \leq U \end{cases}$$

The trapezoidal distribution can be employed to represent assessments where, apart from upper and lower bounds for X, additional upper and lower bounds M_1 and M_2 for the most likely value are defined. When $M_1 = M_2$, the trapezoidal density reduces to the triangular.

Normal

A random variable X is said to be normally distributed if its PDF is given by:

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-(x-\mu)^2/2\sigma^2\right\}, -\infty < x < \infty$$

The normal (gaussian) density is constructed from two parameters, the expected value μ and the variance σ^2. It is a non–finite density, however the probability that X falls within the range $\pm 3\sigma$ is 0.9973; therefore, practically the bounds are specified. The normal distribution is symmetrical and unimodal, and appears suitable when a

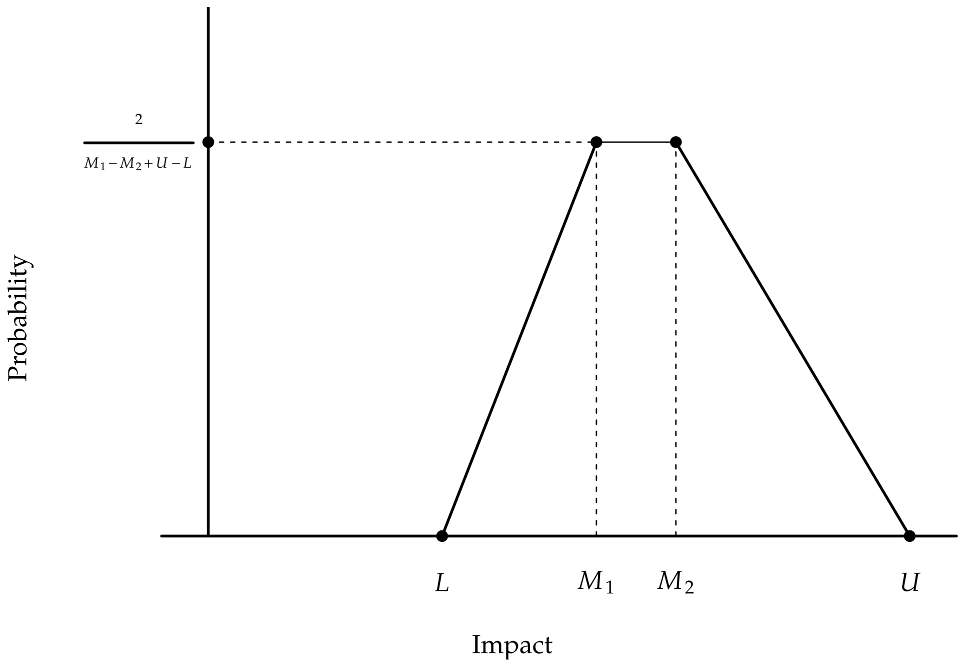

Figure 3.7: The trapezoidal distribution.

single point assessment is at hand. In order to obtain a baseline value for CoV, for comparison with the previous standardised densities, a gaussian with mean $\mu = 1/2$ and range $6\sigma = 1$ can be assumed; in this case, it can be readily shown that $CoV = 1/3 \approx 0.33$.

Log–normal

A random variable X follows a log–normal distribution if its PDF is given by:

$$f_X(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left\{-(\ln x - \mu)^2/2\sigma^2\right\}, x > 0$$

where μ and σ^2 are the mean and variance of the normally distributed random variable $Y = \ln X$. When historical data are available, the log–normal provides a good fit [Wal97], [Yan05], since these data are lower bounded and positively skewed. The formulation of a log–normal distribution for modelling task durations, using a base estimate, a contingency amount, and an overrun probability assessment has been suggested [KI07]; for cost modelling, a similar application can be found in [PS06]. The distribution is unimodal, has lower bound at zero, and no upper bound. The expected value and the variance of the X are given by:

$$\mathrm{E}[X] = e^{\mu + \sigma^2/2}$$

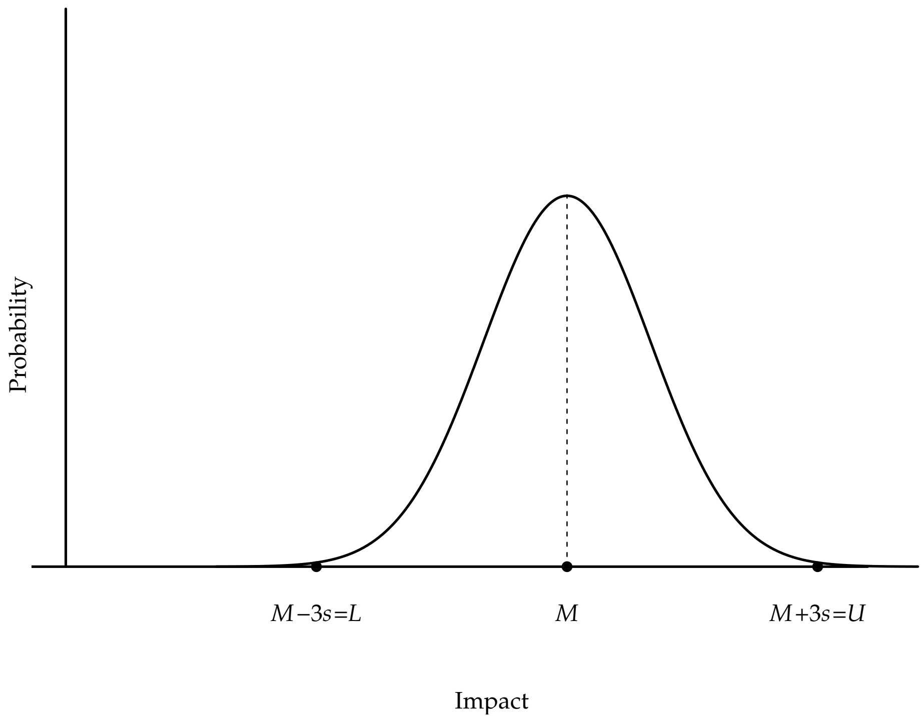

Figure 3.8: The normal distribution.

$$\mathrm{Var}[X] = (e^{\sigma^2} - 1)e^{2\mu+\sigma^2}$$

Since, in this context, the densities are constructed upon expert opinions, the mean μ_0 and variance σ_0^2 of the target distribution are specified. Then, the log–normal density can be defined with parameters:

$$\mu = \mathrm{E}[\ln X] = \frac{1}{2}\ln\left\{\frac{\mu_0^4}{\mu_0^2 + \sigma_0^2}\right\}$$

$$\sigma^2 = \mathrm{Var}[\ln X] = \frac{1}{2}\ln\left\{\frac{\mu_0^2 + \sigma_0^2}{\mu_0^2}\right\}$$

Weibull

The Weibull distribution is a flexible left–bounded distribution. It has seen a widespread adoption in uncertainty modelling over the last years, as a generalisation of the exponential distribution, yet in cost engineering its use is rather rare as of today.

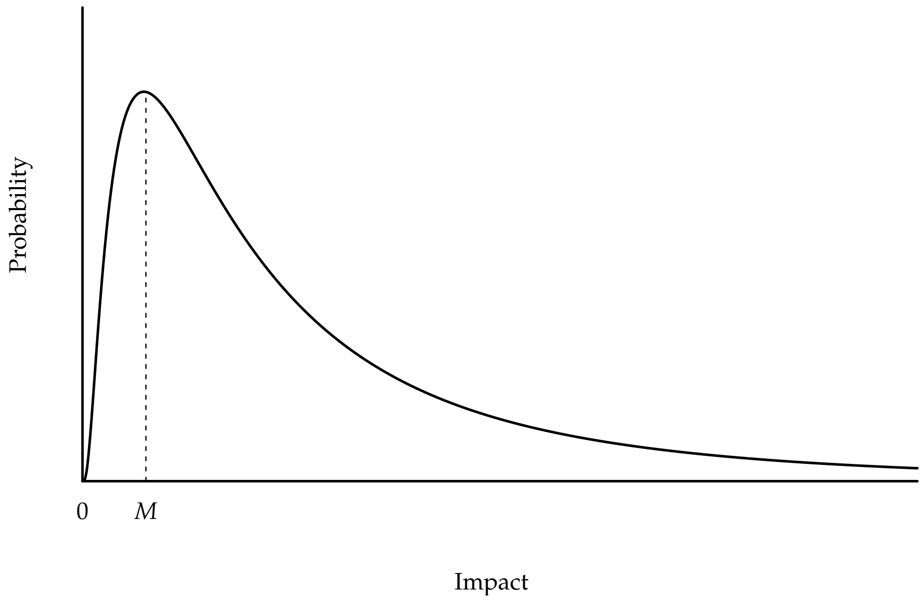

Figure 3.9: The log–normal distribution.

Finally, other proposed families include the following: gamma, Pareto, inverse gaussian, exponential inverse gaussian, Burr, etc.

3.5 Selecting a univariate distribution

The problem of choosing appropriate statistical distributions for the representation of individual uncertain costs has generated much controversies. Firstly, the selection between bounded and unbounded densities has been at the epicenter of this debate. A basic argument against closed–end distributions is that they may exhibit overconfidence owing to their bounded form or, in other words, they do not allow extreme "unexpected" values. However, this feature can prevent from introducing unrealistic extreme values when the bounds are well–determined. Besides, the assessed bounds can be conservatively revised in the presence of high uncertainty, and hence produce safer ranges. Finally, cost overruns in construction projects are rather explained by the occurrence of unforeseen rare events than by slightly deficient tail modelling of the already assessed risks [Pal12], [RB04], [IN09], [HP98]. Hence, in the present study, closed–end densities are considered as more appropriate.

With regard to the choice between the uniform, the triangular and the beta distribution, the three more representative bounded densities, illustrating facts may be found e.g. in the works of [Joh97], [vDK02], [Yan05], [FS03], [KLS$^+$06]. In the previous section, the main characteristics of the former two have been exposed. The triangular and the beta distributions, when fitted on symmetrical judgements, yield likewise symmetrical densities [JZJ03].

The beta distribution was chosen in the present work for the representation of individual risks. The beta density family indeed complies to the generic requirements discussed in Section (3.4), as well as to the specific characteristics that have emerged throughout the actual risk analysis described in the case study. Beta densities can be unimodal, they have a finite range, and can be modelled from symmetric to highly skewed, allowing a high degree of flexibility [GF00]. In the following section, the theoretical background for the beta distribution family is exposed.

3.6 The beta distribution

The beta distribution is a closed–end distribution widely used in probabilistic modelling. The PDF[3] of the *standard* beta distribution, bounded in $[0,1]$ is given by the formula:

$$f(z;\alpha,\beta)=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}z^{\alpha-1}(1-z)^{\beta-1}, 0\leq z\leq 1 \tag{3.2}$$

where $\alpha,\beta>0$ are the shape parameters, and Γ is the gamma function defined for complex numbers with positive real part:

$$\Gamma(s)=\int_0^\infty u^{s-1}e^{-u}\,\mathrm{d}u$$

The density can be written also as:

$$f(z;\alpha,\beta)=\frac{1}{\mathrm{B}(\alpha,\beta)}z^{\alpha-1}(1-z)^{\beta-1}, 0\leq z\leq 1 \tag{3.3}$$

where B is the beta function (also called the Euler integral of the first kind), defined for complex arguments with positive real part:

$$\mathrm{B}(s,t)=\int_0^1 u^{s-1}(1-u)^{t-1}\,\mathrm{d}u$$

For $\alpha>1,\beta>1$ the PDF is unimodal, and for $\alpha=\beta$ is symmetric about the vertical line $z=1/2$. The expected value and the variance are, respectively:

$$\mathrm{E}[Z]=\frac{\alpha}{\alpha+\beta} \tag{3.4}$$

[3]The CDF of the beta distribution has no closed analytical expression [Gar99].

$$\mathrm{Var}[Z] = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} \tag{3.5}$$

Obtaining an upper bound for the variance in the unimodal case, is straightforward; when $\alpha > 1$ and $\beta > 1$:

$$(\alpha+\beta)^2(\alpha+\beta+1) > (2\sqrt{\alpha\beta})^2(1+1+1) = 12\alpha\beta$$

From Equation (3.5) it follows that $\mathrm{Var}[Z] < 1/12$.

For the median, no closed form exists. In the unimodal case $\alpha > 1, \beta > 1$, solving $\mathrm{d}f(z)/\mathrm{d}z = 0$ for z, yields the mode (global maximum) of the density m:

$$m[Z] = \frac{\alpha - 1}{\alpha + \beta - 2} \tag{3.6}$$

If the coefficient of variation is denoted by $CoV = c$, the following formula can be derived from Equations (3.4), (3.5):

$$c^2 = \frac{\beta}{\alpha(\alpha+\beta+1)} \tag{3.7}$$

When Z follows the standard beta distribution, then the random variable X defined through the linear transformation:

$$X = L + (U - L)Z \tag{3.8}$$

is bounded in the interval $[L, U]$. Then X is said to follow the *generalised beta* distribution. After elementary calculation, the PDF of X is:

$$f(x; \alpha, \beta, L, U) = \frac{1}{\mathrm{B}(\alpha,\beta)} \frac{(x-L)^{\alpha-1}(U-x)^{\beta-1}}{(U-L)^{\alpha+\beta-1}}, L \leq x \leq U \tag{3.9}$$

The expected value, the variance and the mode of the (unimodal) generalised beta are readily obtained:

$$\mathrm{E}[X] = (U - L)\frac{\alpha}{\alpha+\beta} + L \tag{3.10}$$

$$\mathrm{Var}[X] = (U-L)^2 \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} \tag{3.11}$$

$$M[X] = (U - L)\frac{\alpha - 1}{\alpha + \beta - 2} + L \tag{3.12}$$

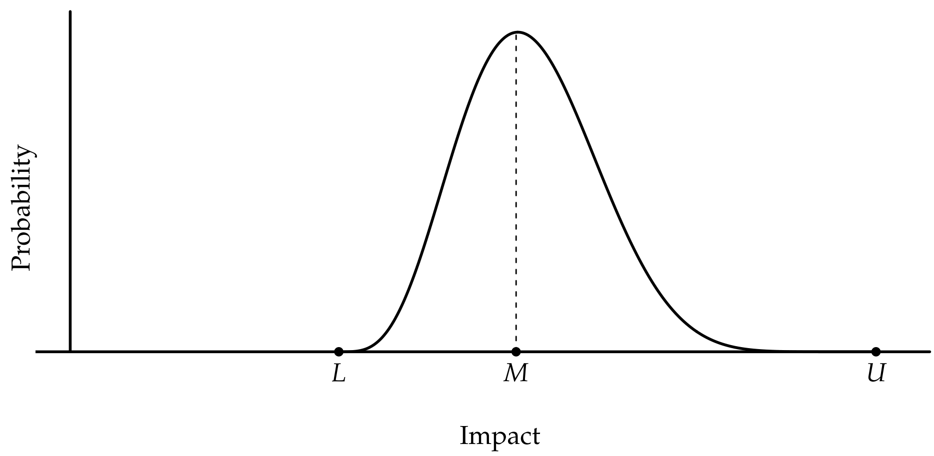

Figure 3.10: The generalised beta distribution.

3.7 Proposed implementation based on the beta distribution

The construction of beta density requires the assessment of four parameters. In the analysis developed for the case study in the present work, the experts were asked to assess a triad comprising the optimistic, most likely and pessimistic estimates, L, M and U, respectively. The "missing" fourth parameter has long been a subject of controversy, having also given ground for interesting alternative developments. To that end, a proposed methodology is presented in the subsequent paragraphs, in order to make use of an—otherwise lost—additional element of information.

In particular, the estimation of the two shape parameters α, β that complete the construction of the beta density is based on a novel view of the concept of confidence in assessment. Firstly, one can observe that the confidence in the assessed mode is inversely related to the density's dispersion. Hence, the coefficient of variation, as dimensionless measure, makes an appropriate candidate to express this concept.

In the following it is assumed that the assessed uncertainty is a risk (has a positive impact). For an opportunity O_j (uncertainty with negative impact) the same methodology applies to the risk $R_j = -O_j$, where the use of the minus is symbolic. For the coefficient of variation of the generalised beta c_1 it follows from Equations (3.10) and (3.11) that:

$$c_1^2 = \frac{(U-L)^2 \alpha\beta}{(U\alpha + L\beta)^2(\alpha+\beta+1)} \tag{3.13}$$

Using Equation (3.7), the ratio between the CoV of the generalised beta bounded

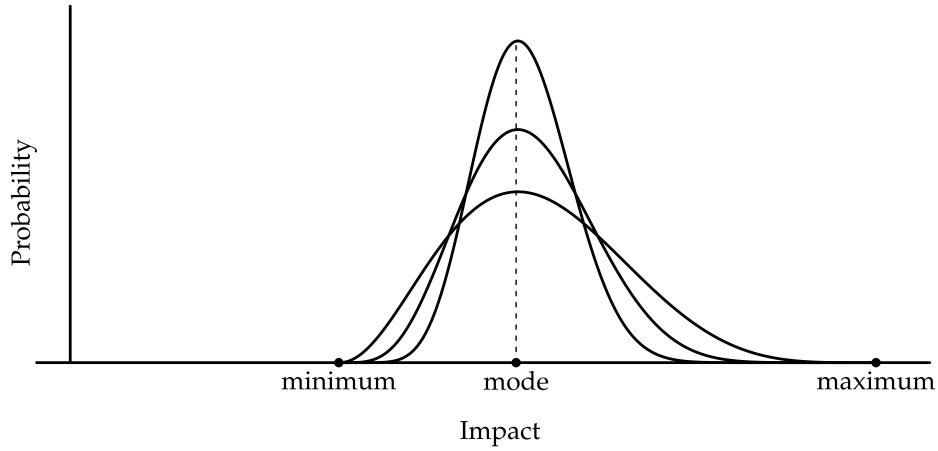

Figure 3.11: The beta distribution with low confidence level (lower curve), moderate level (middle curve) and high level (upper curve).

in $[L, U]$ and the CoV of the standardised beta bounded in $[0, 1]$ can be written as:

$$\frac{c_1}{c} = \frac{U\alpha - L\alpha}{U\alpha + L\beta} \tag{3.14}$$

From Equations (3.7) and (3.13) it is clear that c_1, unlike c, depends on the two bounds L, U. Equation (3.14) implies that $c_1 \leq c$. When $L > 0$, it holds for c_1:

$$\frac{\partial c_1}{\partial L} = -c \cdot \frac{U\alpha(\alpha + \beta)}{(U\alpha + L\beta)^2} < 0$$

Therefore c_1 is decreasing in L; when $L = 0$, c_1 attains its maximum value $c_1 = c$, and it is independent from the bounds. These observations indicate that the coefficient of variation of the standardised beta can be selected to represent the lack of confidence in assessment: it is a dimensionless dispersion measure, it does not depend on the bounds, and it can be directly assessed, as will be shown in the remainder section.

Next, the levels of lack of confidence expressed as c–values need to be numerically specified. It was shown in Section (3.4) that the uniform distribution bounded in $[0, 1]$ has a CoV value of $\sqrt{3}/3 \approx 0.58$; this can be viewed as the theoretical maximum. It will be shown that the proposed method for constructing subjective densities satisfies indeed this boundary condition, and can attain values as close to 0.58 as desired. The standardised normal, which has a CoV value of $1/3 \approx 0.33$, can be thought of as a baseline value. When large uncertainties are involved, the density should be flatter then the normal [JZJ03]; in other cases the dispersion expressed by the normal density may be conservative. Therefore, the values $c = 0.25, 0.33, 0.42$ for the standardized beta were selected to represent respectively high, moderate and low confidence in assessment. A comparison of the three levels can be depicted in Figure (3.11).

Figure 3.12: Indirect elicitation of the fourth parameter of the beta density, by the subjective confidence percentage, according to Formula (3.15).

A more generic approach can be taken by asking the expert to provide a number k in $[0,1]$ (equivalently, a percentage $100 \cdot k\%$ in $[0,100]$) in order to express the confidence in the assessment. This possibilistic quantity can be translated into the coefficient of variation c according the rule:

$$c = 0.18(1-k)(3.2+k) \tag{3.15}$$

The relation is graphically shown in Figure (3.12). The levels $k = 0\%, 50\%, 100\%$ correspond to $c = 0.58, 0.33, 0$ as desired with respect to the boundary and baseline values discussed above.

In some cases, the assessment (L, M, U) can be highly skewed. When the modal value M is relatively very close to one of the two bounds, the probabilistic expression of the cost element significantly departs from statistical normality, implying either a poor assessment or large epistemic uncertainty. Such a case is inconsistent with high confidence, and results to an overly thin distribution tail. For instance, if M is relatively very close to L, then a beta density will have a very thin right tail, and a significant part of the cost range on the right will be almost ignored. Therefore, in the present study it is advised to assume an increased lack of confidence value c', according to the following formula:

$$c' = \frac{(\sqrt{3}c)^{\psi}}{\sqrt{3}} \tag{3.16}$$

where ψ is a measure of skewness, given by:

$$\psi = 2\left(\frac{M-L}{U-M} + \frac{U-M}{M-L}\right)^{-1}$$

When $M \to L$ or $M \to U$, it follows that $\psi \to 0$ and c' attains its theoretical maximum $\sqrt{3}/3$ (see the following proposition). When $M \to (L+U)/2$ (symmetry), then $\psi \to 1$ and $c' \to c$.

The formulation of the previous arguments is condensed in the following:[4]

Proposition. Suppose that the lower bound L, the upper bound U and the modal value M of a beta density are assessed, $L < M < U$. Moreover, suppose that the dispersion of the desired density (lack of confidence) is evaluated through the assessment of the coefficient of variation c of the corresponding standardised beta, where $c < \sqrt{3}/3$. Then, the two shape parameters α, β of the generalised beta density are given by the formulas:

$$\alpha = \frac{-\{c^2(3m-1)+m-1\}+\sqrt{D}}{2c^2} \tag{3.17}$$

$$\beta = \frac{(1-m)\alpha + 2m - 1}{m} \tag{3.18}$$

where m and $D > 0$ are given by:

$$m = (M-L)/(U-L) \tag{3.19}$$

$$D = (3m-1)^2c^4 + 2(3m^2-1)c^2 + (m-1)^2 \tag{3.20}$$

Moreover, the shape parameters calculated through the Formulas (3.17), (3.18) always yield a unimodal density. Finally, the bound $c = \sqrt{3}/3$ is best possible.

Proof. From Equation (3.8), it yields that the modal values m and M of the standardised and generalised density, respectively, are related as:

$$m = (M-L)/(U-L) \tag{3.21}$$

[4] The main result of the proposition has also appeared in [Tam11] without proof.

Once the mode m of the standardised beta is calculated as above, the relation with the distribution parameters α, β can be written according to Formula (3.6) as:

$$m = \frac{\alpha - 1}{\alpha + \beta - 2} \tag{3.22}$$

which can be written also as:

$$m(\alpha + \beta + 1) = \alpha - 1 + 3m \tag{3.23}$$

and

$$m\beta = \alpha(1 - m) + (2m - 1) \tag{3.24}$$

Equation (3.7) using the above two forms can be written as:

$$c^2 = \frac{\beta}{\alpha(\alpha + \beta + 1)} = \frac{m\beta}{\alpha m(\alpha + \beta + 1)} = \frac{\alpha(1 - m) + (2m - 1)}{\alpha(\alpha - 1 + 3m)}$$

which leads to a quadratic equation for α:

$$Q = c^2\alpha^2 + \{c^2(3m - 1) + m - 1\}\alpha + (1 - 2m) = 0$$

The discriminant of the equation $Q = 0$ is:

$$D = (3m - 1)^2c^4 + 2(3m^2 - 1)c^2 + (m - 1)^2$$

which can be rewritten as:

$$D = \{(3m + 1)c^2 + (m - 1)\}^2 + 4mc^2(1 - 3c^2) \tag{3.25}$$

Given that $c < \sqrt{3}/3$, it follows that $4mc^2(1 - 3c^2) > 0$, therefore the above relation implies that $D > 0$. Hence, the quadratic equation $Q = 0$ has two real roots. If α denotes the larger of these roots, then:

$$\alpha = \frac{-\{c^2(3m - 1) + m - 1\} + \sqrt{D}}{2c^2}$$

Hence, the requirement $\alpha > 1$, in order to obtain a unimodal density, is equivalent to:

$$\sqrt{D} > c^2(3m + 1) + (m - 1) \tag{3.26}$$

By Equation (3.25) it follows that (3.26) is satisfied, and therefore $\alpha > 1$. The second shape parameter β can be derived from (3.24), and can be also shown to satisfy the condition for unimodal beta:

$$\beta = \frac{(1-m)\alpha + 2m - 1}{m} > \frac{1 - m + 2m - 1}{m} = 1$$

The boundary condition $c_0 = \sqrt{3}/3$ is indicated by the fact that c_0 is the maximum possible dispersion as the CoV of the standard uniform distribution. For $c = \sqrt{3}/3$, and selecting $m = 1/3$, the above calculation yields $\alpha = 1$, hence no unimodal solution. When $c > \sqrt{3}/3$, then there exists an $\epsilon > 0$ such that $c^2 = 1/3 + \epsilon$. By selecting again $m = 1/3$ it is obtained that $D = -4\epsilon/3 < 0$, hence no solution in terms of α and β. Therefore, $c < \sqrt{3}/3$, and the bound is best possible, since it always yields a unimodal solution.

This condition is marginally restrictive within the unimodal class of the beta distribution family, since for $\alpha \geq 3$ it holds that $\alpha(\alpha+\beta+1) \geq 3\beta + 3(\alpha+1) > 3\beta$. Therefore, from Formula (3.7), $c^2 = \beta/(\alpha+\beta+1) < 1/3$, thus $c < \sqrt{3}/3$. Only in the case when $\alpha < 3$ and $\beta > \alpha(\alpha+1)/(3-\alpha)$, the coefficient of variation c can exceed the value of $\sqrt{3}/3$, so this small fraction of beta densities are not obtained by the proposed construction. If $\alpha < 3$ and it is set $\alpha = 3 - \delta$ where $0 < \delta < 2$ then, from Formula (3.6), it is derived that the lower undesired value for β, $\beta = (3-\delta)(4-\delta)/3\delta$ yields the greater possible "problematic" m value, $m = (\alpha - 1)/(\alpha + \beta - 2) = \delta/6 < 1/3$, indicating particularly skewed densities.

□

The same procedure can be applied also in single point assessments. If a symmetrical density is desired, then $\alpha = \beta$, therefore three parameters are required. In order to ensure consistency with the previous approach, these values can be chosen to be: (1) the single point estimate M, (2) the lack of confidence in assessment c, and (3) the range R_d of the density. Then $L = M - R_d/2$, $U = M + R_d/2$; Formula (3.19) yields:

$$m = \frac{M - (M - R_d)}{(M + R_d) - (M - R_d)} = \frac{1}{2}$$

Then, from Formula (3.20):

$$D = \left(\frac{1 - c^2}{2}\right)^2$$

From (3.17) and (3.18) the shape parameters are obtained:

$$\alpha = \beta = \frac{1}{2}\left(\frac{1}{c^2} - 1\right) \tag{3.27}$$

Confidence	c	$\alpha = \beta$
perfect	0	∞
high	0.25	7.5
moderate	0.33	4.0
low	0.42	2.4
infimum	0.58^{-}	1^{+}

Table 3.1: Beta parameters for the three selected levels of lack of confidence in assessment, symmetrical case.

Condition $c < \sqrt{3}/3$ again directly implies that $\alpha > 1, \beta > 1$. Table (3.1) shows the shape parameters corresponding to the selected confidence levels.

It is worth noting that, under the assumptions of the developed methodology, the selection of a range equal to six times the standard deviation, akin to the gaussian, implies that $c = 1/3$, which is indeed the CoV of the standard normal distribution. Thus, replacing the gaussian with a symmetrical beta density is consistent, not only in terms of dispersion, but also with the range requirements. In order to prove that, Equation (3.14) setting $\alpha = \beta$ yields:

$$c_1 = \frac{s_d}{M} = \left(\frac{U - L}{U + L}\right) c$$

Replacing $L = M - R_d/2$, $U = M + R_d/2$, this relation is written as:

$$s_d = \frac{cR_d}{2}$$

From the above formula it is readily seen that $c = 1/3$ if and only if $R_d = 6s_d$.

3.8 Criticism of beta distribution

The beta distribution, despite its wide use, has been criticised mainly because:

1. It suffers from difficulties in maximum likelihood estimation.

2. Its shape parameters do not seem to have a clear interpretation.

3. Its construction requires four numbers in contrary to the commonly assessed three (minimum, mode, maximum).

4. The simulation is relatively slow.

These points of criticism need to be addressed; the devised and proposed method for constructing beta densities from expert judgements has the following characteristics:

1. The PDF's are not established based upon historical data, therefore the maximum likelihood estimation is not relevant.
2. The shape parameters are not directly elicited from experts, but mathematically derived from explicit meaningful judgements.
3. The fourth required parameter is utilised as an additional tool to allow for flexibility and incorporation of information that is otherwise lost.
4. The simulation speed issue is not negligible, yet rather obsolete, considering the power of modern computers.

3.9 Other implementations using the beta distribution

The beta distribution is at the core of PERT (Program Evaluation and Review Technique) approach [MRCF59]. According to the "classical" PERT procedure [Wil05], [PPR99] the mean and the variance are computed through the approximate formulas:

$$\hat{\mu} = \frac{1}{6}(L + 4M + U)$$

$$\hat{\sigma}^2 = \left\{ \frac{1}{6}(U - L) \right\}^2$$

The shape parameters α, β can be analytically obtained, or estimated through various methods [Yan11], [KLS+06], [Wil05], [VPvD11]. The usefulness of the above formulas is usually stressed by the fact that the construction of the generalised beta is possible by use of three parameters, while four of them are generally required. However, the validity of the above procedure has been long criticised [vDK02]; for instance, the variance is constant, conditional only on the range $U - L$ [Hah08] and no consideration whatsoever for confidence is contained.

Other techniques have gone beyond the PERT limitation. By eliciting estimates of a fourth point on the desired density, the two shape parameters α, β can be calculated directly, without relying on assumption of a standard deviation equal to 1/6 of the range. In case the fourth element is selected to be the ninetieth percentile value, the two shape parameters are obtained by solving a cubic equation [GF00]. Several methods for constructing beta distribution can be found [KLS+06] involving the endpoints and two additional statistical characteristics: (1) the mean and the variance, (2) the mean and the mode, (3) the mode and the variance, (4) the mode and an arbitrary quantile, and (5) two quantiles. However none of these alternatives appears practical in the present context. Hahn [Hah08] proposed a mixture distribution based on the beta, which additionally employs an "uncertainty weight" parameter, in line with the ideas developed in the present work.

3.10 Alternative approaches

Several scholars have tried to go beyond the beta distribution representations. For instance, Johnson [Joh97] provided an empirical method to approximate the beta distribution with a triangular one. The method is based on the approximation of the three triangular distribution parameters with linear expressions of three standard beta distribution quantiles (two extreme quantiles and the median), derived by linear regression from a representative sample. This approach allows for constructing a triangular distribution directly from assessed quantiles.

Lau and Somarajan [LS95] supported the use of the four parameter Ramberg–Schmeiser (generalised lambda) distribution in place of the beta, while van Dorp and Kotz [vDK02] investigated the two–sided power distribution as a meaningful alternative to the beta for modelling uncertain time in the context of decision analysis. Jiang et. al [JZJ03] proposed a model with unimodal density and one additional parameter to provide flexibility. Elkjaer [Elk00] suggested the use of Erlang–k distribution.

Despite the widespread use of probabilistic methods, it has been questioned in various studies (e.g. [TBF99]) whether the frequentistic interpretation of probability is advantageous in the context of expert judgement over the classical one. An often stated argument against probabilistic approaches for first–of–a–kind projects is based on the lack of previous data [DBH07]. Next to that, it has been argued whether standard probability theory can deal with information described in natural language. In an attempt towards a generalised theory of uncertainty, Zadeh[5] [Zad06] argues that not only information is statistical in nature but, more generally, information is a constraint with statistical uncertainty being a particular case. These considerations have given ground to the fuzzy logic approach as an extension to

[5]Zadeh is credited with developing fuzzy set theory.

bivalent logic. Efforts to exploit fuzzy logic within the Civil Engineering domain are listed in [CT01]. For an overview on the use of fuzzy risk assessment methodology one may refer to [DBH07].

Moreover, interval analysis, grey numbers and random sets [BT10] have been proposed by several scholars as being practical and efficient in cost engineering. Instead of statistical distributions, individual cost elements can be represented by intervals [PS06]. The Dempster–Shafer Theory of evidence, also known as the theory of belief functions, introduced by Shafer in 1976 as a generalization of the Bayesian theory of subjective probability [VLDD$^+$12], has seen some interesting implementations over the recent years. Finally, artificial neural networks or multiple regression analysis to model expert judgements resulting from multiple influence factors have also been employed [ATKFA97].

While the aformentioned techniques can offer useful tools for dealing with uncertainty in a variety of engineering problems, the author believes that they do not offer real advantages over the probabilistic analysis. Firstly, handling dependence among risks is rather complicated. Moreover, they do not constitute a transparent framework to be used by individuals with little or no significant background on the subject. Finally, the automation of the whole process is very demanding.

Chapter 4

Multivariate dependence

The present chapter is organised as follows. Section (4.1) introduces the problem of cost aggregation under dependence. In Section (4.2) independence is defined and discussed, as the first step to address the problem. Negation of independence leads to the concepts of causation and covariation, explained in Section (4.3). Section (4.4) outlines the typical tools to capture pairwise dependence, while the relation to information sources is examined in Section (4.5). Section (4.6) proceeds to the multivariate context, where a multitude of variables is involved. Two different dependence structures are studied in Section (4.7). Finally, in Section (4.8) the gaussian copula simulation technique is described.

4.1 Introduction and general formulation

As stressed in Chapter 2, the calculation of the total risk cost of a project involves several variables, i.e. the individual cost elements $R_j, j = 1, ..., n$ which represent the identified risks. Once these items are modelled, e.g. by probability densities, as described in Chapter 3, the risk term R of the total cost can be expressed as follows:

$$R = \sum_{j=1}^{n} R_j$$

There are two basic problems associated with this formulation. Firstly, since $R_j, j = 1, ..., n$ are random variables, the above aggregation is not straightforward as numerical summation. The total risk R is a random variable itself which, in the general case, does not belong to a particular statistical distribution family. Therefore, the probability density function of R cannot be directly constructed; this is usually achieved through a simulation technique, e.g. a Monte Carlo method[1]. The second and most important problem lies in the possible dependence among the variables $R_j, j = 1, ..., n$. Dependence needs to be detected, measured and integrated into the simulation process, making the computational task highly challenging.

In particular, in a Monte Carlo approach, a sufficiently large (dictated by the desired output precision) number m of random n–tuples $\mathbf{r}^i = (r_1^i, ..., r_n^i), i = 1, ..., m$ are generated in order to simulate the random vector $\mathbf{R} = (R_1, ..., R_n)$. Then, these variates are combined in order to yield summary statistics for R. If the components $r_j^i, j = 1, ..., n$ of every i–iteration are generated independently, it is implied that the random variables $R_j, j = 1, ..., n$ are being assumed independent. However, in general, some kind and degree of dependence is present among the variables, hence the above simulation has to include additional considerations, examined throughout the remainder chapter.

In order to address these matters, the concepts of independence and dependence among the input random variables need to be addressed. This is the purpose of the forthcoming sections, wherein these two notions are discussed. Moreover, methods to measure, implement and propagate dependence into the simulation process are gleaned from the literature. Focus is directed to practicability, efficiency, and retention of the relevant calculus to a reasonable level. In this context, the strive for scientific validity may easily lead to extremely complicated and non–appealing techniques, while the use of overly complex dependence structures may act against viewing, understanding, and control. On the downside, oversimplifications pose a threat to the scope and efficiency of the developed ideas.

4.2 The concept of stochastic independence

Let two events E and F, defined on a common probability space $(\Omega, \mathcal{E}, \mathbb{P})$, with $P(E) \neq 0$ and $P(F) \neq 0$. The *conditional* probability $P(E|F) = P(E \cap F)/P(F)$ expresses the likelihood that event E occurs, given that event F has occurred, i.e. the fact that the occurrence of F may affect the occurrence of E; in this case the probability of E changes from $P(E)$ to $P(E|F)$. Generally, these two values are different; however case may be that $P(E|F) = P(E)$. Then, event E is called *independent* of F. Likewise, event F is called independent of E when $P(F|E) = P(F)$. By virtue of the above definition of conditional probability, both cases are proved to be equivalent to the following condition[2]:

[1] An outline of Monte Carlo technique is given in Appendix B.

[2] Also referred to as *the multiplication law of independence.*

Definition: A pair of events E and F is called independent if:

$$P(E \cap F) = P(E) \cdot P(F)$$

When (E, F) represents a pair of independent events, then independence also holds for the pairs (E, F^c), (E^c, F) and (E^c, F^c), where A^c denotes the complement of an event A.

If two events are independent, it does not mean that they are disjoint. Disjoint events are mutually exclusive, i.e. their intersection is null. In this case, $P(E \cap F) = 0$. Independent events can occur simultaneously, but the occurence of one event does not influence the occurrence of the other.

It is necessary and possible to extend the above definition to an arbitrary number of events. In fact, the $n > 2$ events $E_j, j = 1, ..., n$ are called (mutually) independent if, for any k–subset of the form $\{j_1, ..., j_k\} \subset \{1, ..., n\}$, the following property holds:

$$P(E_{j_1} \cap ... \cap E_{j_k}) = P(E_{j_1}) \cdot ... \cdot P(E_{j_k}) \tag{4.1}$$

The number of such combinations is:

$$\binom{n}{2} + ... + \binom{n}{n} = \sum_{k=0}^{n} \binom{n}{k} - \binom{n}{0} - \binom{n}{1} = 2^n - n - 1$$

It is important to require the multiplication rule for all possible combinations; as typically stated "pairwise independence does not imply mutual independence" [Has03]. Furthermore, independence is not transitive: if X is independent of Y, and Y is independent of Z, then X and Z are not necessarily independent. Pairwise independence represents indeed only a small fraction of the whole dependence structure, since the total number of pairs is $n(n-1)/2$, while the number $2^n - n - 1$ of all Equations (4.1) grows exponentially with n. Therefore, elementary as it may be to mathematically define, independence is practically difficult to check on data.

Independence for a finite set of random variables can be defined by means of the sets of the form $S = \{X \leq a\}$. The random variables $X_j, j = 1, ..., n$ are independent if and only if for every $(a_1, ..., a_n)$, $\{X_j \leq a_j\}, j = 1, ..., n$ are independent events. In general, independence of random variables can be defined upon the independence of all events of the type $\{X \in S\}$, where S stands for any Borel set on the real line.

Independence between random variables is a very convenient property. If the variables X and Y are independent, it can be shown that [DKLM07]:

- The expectation operator is multiplicative:

$$\mathrm{E}[X \cdot Y] = \mathrm{E}[X] \cdot E[Y] \tag{4.2}$$

- The variance operator is sum–preserving:

$$\mathrm{Var}[X + Y] = \mathrm{Var}[X] + \mathrm{Var}[Y] \tag{4.3}$$

- The joint CDF equals to the product of the marginal CDF's:

$$F_{XY}(x, y) = F_X(x) \cdot F_Y(y) \tag{4.4}$$

- The joint PDF equals to the product of the marginal PDF's:

$$f_{XY}(x, y) = f_X(x) \cdot f_Y(y)$$

Similar expressions can be obtained for $n > 2$ random variables. Each of the last two properties is sufficient to imply independence between X and Y. Moreover, it is clear from those two properties that the multivariate problem of calculating joint distributions degenerates to a mere multiplication of marginal distributions under the independence assumption. However, the property:

$$\mathrm{E}[X + Y] = \mathrm{E}[X] + \mathrm{E}[Y] \tag{4.5}$$

always holds, regardless of dependence among the random variables. Therefore, information on the expected value of the sum of random variables does not update when additional information on the dependence among them is obtained.

Turning to the context of probabilistic cost analysis, for $n > 2$ risk impacts $R_j, j = 1, ..., n$, linearity of expectation operator yields:

$$\mathrm{E}[R] = \sum_{j=1}^{n} \mathrm{E}[R_j] \tag{4.6}$$

Since the present study primarily focuses on the estimation of "safe" (upper quantile) values for the total cost, average values are of no particular interest, since they possess high probability of exceedance. The "law of averages", expressed by Formulas (4.5) and (4.6), can falsely lead to the "flaw of averages" [Sav09], where erroneous broader decisions within a system are being made upon individual expectations on the components. Moreover, the average value of the total cost does not include considerations for individual cost dispersions. Finally, average values do not account for dependence. This aspect is important, as shown in the forthcoming section.

4.3 From causation to covariation

The study of dependence is of paramount importance in cost engineering. Firstly, albeit dependence considerations do not affect the average value of the total risk cost—as shown in the previous section—they can decisively modify the “safe” economic values used in the analysis. Besides, dependence among project activities can alter the schedule, imposing indirect costs, mainly due to resulting delays and increased variability [Wan02].

Several researchers have argued that dependence is even more important than the selection of individual distributions [CYC09], [OO05], [Pet12]. Apart from the quantitative aspect, the detailed study of already realised project phases or similar undertakings through backwards (diagnostic) reasoning in terms of dependence, can reveal facts and effects useful for performing direct (predictive) inference [Sha04]. Therefore, a dependence–aware approach opens the door to a more realistic, efficient and dynamic risk management.

In a multivariate framework, uncritically omitting[3] dependence can be justified only upon the desire for computational tractability [Wan98]; the independence assumption is a specific, restricting and usually unrealistic modelling choice, since it ignores relationships between events and effects. Furthermore, it can have a significant influence on cost estimates, giving rise to serious divergences from the actual cost [Wal97], [BM98], [ABP+06].

Typically, the first step in order to describe dependence between risks, is the attempt to model cause–and–effect relations to represent the perceived underlying associations. A cause can be referred to as *risk factor*, while the effect as *risk symptom* [CT01]. This approach reflects the perspective that risk can be deterministically predicted once sufficient information is available. There are two critical facts indicating the physical constraints in this treatment: (1) complete knowledge is unreachable, therefore uncertainty cannot be eliminated [CLP07], especially for future events [Ave11] and (2) modelling functional relations is typically impractical [FNH+04]. The above described attitude is congruent to the propensity interpretation of randomness, introduced by C.S. Peirce and further developed by K. Popper [Cor05], where probabilities result from underlying causal mechanisms.

The deterministic way to interpret effects, natural as it may appear within a time sequence of activities, can often lead to the *post hoc ergo propter hoc* fallacy [BK06, p. 148], or *illusory correlation* [GW04]. In particular, the analyst may

[3] As expressed by H. Putnam, all facts are fallible, but questioning them requires a counter argument, i.e. another fact [PGW08].

be tempted to attribute the appearance of an event entirely to the influence of preceding events, based rather on its position in the broader time sequence, or on personal arbitrary preconceptions, than on objectively specific evidence. Moreover, the magnitude of dependence is typically overestimated when primarily based on a presumed causal theory [Sch04]. In fact, the interrogation format, due to bias interfering with frequentistic and causal perceptions of risks, can influence reasoning. Although dependence among possible effects can sometimes be designated as either causal or merely associative when "hard" data are available [BE09], or extracted from frequently observed patterns [HKP05], in an expert judgement context this distinction is hardly evident [Ave11].

The need to relax the deterministic assumption of strict cause–and–effect relations, yet still be able to model dependence, leads to the concept of *covariation*. This transition somehow concurs with the shift from risk to uncertainty, described in Section (2.3). Although causation and covariation are often presumed identical, they fundamentally differ: for instance, covariation is indeterminate with respect to direction[4], while causal judgements are very sensitive to the order in which information is presented to the expert [Sha04]. The remarkable work of Cheng [Che97] offers a detailed theoretical review of the subject.

On the one hand, several studies [MS10], [LX03], [CYC09] have supported that a purely statistical treatment of dependence can neither provide insight about dependencies, nor enable the use of available knowledge about causal structures. On the other hand, a purely deterministic description of association is impractical, as previously discussed. Therefore, it is important that the modeller acknowledges that a monolithic approach of dependence can impose undesirable limitations to the scope of the study. To that end, numerous definitions aiming to offer a normative description of dependence have been developed in the literature (correlation, concordance, quadrant dependence, association, stochastic ordering, etc.) [LX03].

Especially in the risk analysis framework, classifications of association types has been attempted [CGRW04], [KC06, p. 16]. In particular, relationship types between activities in construction have been described in [WH00]. The range of different types is obviously dictated by the availability of data; hence, in the present study, such taxonomies have been considered with some care.

A typical dependence type is what is referred to as "common cause failure", where failure in the context of cost estimation may stand for an unexpected excess in cost or time [FHB02]. A "common shock" to the system may lead to failure if the system components possess a perfect positive dependence with respect to

[4]Still, in practice, dependencies are usually depicted by directed arcs.

that common cause [Wil97]. In fact, the project's idiosyncratic characteristics or individual unwanted events are likely to affect multiple cost items [KAE04].

Costs (and durations) in construction can be indirectly correlated if the corresponding activities are performed under similar supervision, site and weather conditions [Yan11]. A typical example in tunnelling is the case of adverse geological conditions, which can affect multiple engineering works of different forms. On the downside, inserting risk factors as zero–cost risks acting as stochastic "switches" within a risk network, requires additional manual effort, making the automation of the process rather complicated. This matters will appear and further discussed in Chapter 5.

4.4 Basic dependence measures

Once dependence is detected, it needs to be measured. Apparently, the simpler way of measuring and describing dependence is by means of scalar indices. A measure[5] δ between two random variables X, Y should satisfy a number of properties [EMS01], [DL09]; e.g. symmetry:

P_1: $\delta(X,Y) = \delta(Y,X)$

It is also desired that δ assumes values in the interval $[-1,1]$:

P_2: $-1 \leq \delta(X,Y) \leq 1$

The bounds ± 1 should be reached when the relationship between X and Y is monotonic:

P_3: $\delta(X,Y) = 1$ if and only if (X,Y) are comonotonic, and $\delta(X,Y) = -1$ if and only if (X,Y) are countermonotonic.

The measure δ should be invariant (up to sign) under monotonic tranformations of the arguments:

P_4: If $t : \mathbb{R} \to \mathbb{R}$ is a strictly monotonic transformation, then:

$$\delta(t(X),Y) = \begin{cases} \delta(X,Y) & \text{, if } t \text{ is increasing} \\ -\delta(X,Y) & \text{, if } t \text{ is decreasing} \end{cases}$$

[5]Different sets of required properties are set for defining measures of *concordance*, *dependence*, or *association*.

Another desired property which, interestingly, contradicts P_4 [DDGK05, p. 247] is the following:

P_5: $\delta(X, Y) = 0$ if and only if X and Y are independent.

In the remainder chapter, the three mostly used[6] scalar measures are presented and discussed, having properties P_1–P_5 as a reference point.

Covariance and linear correlation

Let μ_X and σ_X denote respectively the expected value and the standard deviation of a random variable X. The *covariance* of two random variables X, Y is defined as the expected value of the variable $(X - \mu_X)(Y - \mu_Y)$, namely:

$$cov(X, Y) = \mathrm{E}[(X - \mu_X)(Y - \mu_Y)] \tag{4.7}$$

It is easily shown that:

$$cov(X, Y) = \mathrm{E}[XY] - \mathrm{E}[X]\mathrm{E}[Y] \tag{4.8}$$

From Equation (4.7) it is clear that $cov(X, Y) > 0$ when large values of X tend to coexist with large values of Y, and small values of X with small values of Y. The opposite relation between X and Y is implied when $cov(X, Y) < 0$. From Equations (4.2) and (4.8) it is derived that if X and Y are independent, their covariance is zero. However, the converse is not necessarily true. Covariance satisfies the following property:

$$cov(aX + b, cY + d) = ac \cdot cov(X, Y)$$

The above equation shows that covariance is not suitable to express dependence, since it is not scale–free. The standardised form of covariance is the Pearson's product–moment *linear correlation* coefficient r, defined as:

$$r(X, Y) = \frac{cov(X, Y)}{\sigma_X \sigma_Y}$$

Then, r is invariant (up to sign) under linear tranforms of the variables:

$$r(aX + b, cY + d) = \mathrm{sign}(ab) \cdot r(X, Y)$$

[6] There are many other scalar measures of dependence described in the literature, e.g. in [MK01].

Pearson's linear correlation coefficient indicates the intensity and direction of a linear relation between X, Y; the case $r = 1$ ($r = -1$) describes an increasing (decreasing) perfect linear relationship. As a scalar metric is easily calculated and interpreted; however, it possesses several serious disadvantages, and has often been criticised for its limited worth. For instance, independence between X and Y implies zero correlation; nevertheless, the converse holds only for bivariate elliptical[7] distributions [ELM03], which imposes a rather strict assumption. Likewise, small correlations do not imply weak dependence, hence they cannot be safely ignored. In addition, the following shortcomings of Pearson's correlation coefficient are mentioned, inter alia [BE09], [EMS01], [FNH+04]:

- It detects only linear dependence.
- Feasible correlation values depend on the choice of the marginal distributions; the attainable range can be much tighter[8] than $[-1, 1]$
- Varying correlations in $[-1, 1]$ does not explore possible nonlinear dependencies.
- It is problematic in cases of asymmetric distributions.
- It is not invariant under arbitrary monotone transformations.

Before beginning the quest for a perfect scalar dependence measure, a common drawback of every possible selection should be highlighted. As a single number, a scalar measure cannot express both the type and the magnitude of dependence at the same time. Consequently, given fixed marginal distributions and correlations, there exist infinitely many feasible joint distributions. In fact, this can be easily shown using copula functions. Appendix C contains further details on this matter.

Spearman's rho

The larger the departure of the margins from normality, the more misleading the concept of linear correlation becomes [Jäc02]. It is possible to remedy some of its deficiencies by working with probability–transformed variates. *Spearman's rank correlation* ρ is defined as the Pearson's correlation coefficient of the grades $U = F_X(X), V = F_Y(Y)$:

$$\rho(X, Y) = r(F_X(X), F_Y(Y))$$

[7] Typical members of the elliptical class are the multivariate gaussian and Student distributions.

[8] However, it always holds that $r_{min} < 0$ and $r_{max} > 0$ [Emb09].

Kendall's tau

A pair of observations or independent realisations (x_1, y_1) and (x_2, y_2) of X, Y is called *concordant* (*discordant*) if the observation with the larger value of X has the larger (smaller) value for Y; in other words, when the product $(x_1 - x_2)(y_1 - y_2)$ is grater (smaller) than zero. The population version of Kendall's tau τ is defined as the probability of concordance minus the probability of discordance:

$$\tau(X, Y) = P((X - \tilde{X})(Y - \tilde{Y}) > 0) - P((X - \tilde{X})(Y - \tilde{Y}) < 0) \tag{4.9}$$

where $(\tilde{X}, \tilde{Y})$ is an independent copy of (X, Y).

Both Kendall's tau and Spearman's rho lie in the interval $[-1, 1]$ and they are equal to ± 1 if and only if X and Y are almost surely monotone functions of each other. Therefore, they allow for measuring monotonic and not necessarily linear dependencies between variables. Also they are equal to 0 when X, Y are independent, with the converse not generally true. Kendall's tau and Spearman's rho are distribution–free, i.e. they do not depend on the choice of the marginal distributions, in contrary to the Pearson's correlation. Thus, their assessment can be performed without having necessarily assessed the marginal distributions.

EXAMPLE 4.1: Suppose the value of Kendall's tau is 0.6. Then, Equation (4.9) implies that the probability of concordance is 0.8 and the probability of discordance is 0.2, since their sum is unity. Hence, if a large value for X is observed, then also a large value of Y should be anticipated, with a likelihood of 80%, and not 60% as one might infer from $\tau = 0.6$.

In the present study, it is assumed that the subjectively elicited correlations, are in fact Kendall's tau values. However, the numerical differences with Pearson's linear correlations or Spearman's rho are rather insignificant [FN07], given the degree of epistemic uncertainty. The use of Kendall's tau allows for removing the restrictive assumption of linearity. This fact would make more sense if the subjective data were to be replaced by hard data.

4.5 Dependence and information

Before the quantification of dependence between risks, the following related problems have to be addressed in an expert elicitation process:

- The nature of dependence and the different types of risk association have to be understood by the analyst.

- A coherent and comprehensive set of well–defined questions has to be formulated in order to harvest the most possible information from the experts.

- The number and complexity of questions has to be minimised without compromise in the obtained information.

- A transparent computational framework has to be established, to make use of the assessed information.

Several levels of dependence information can be identified, according to the available sources. In a risk assessment context, however, such as the one described in the case study of the present work, the only sources are the elicited expert judgements, typically in qualitative form. In few cases, also historical descriptive data can be at hand.

Dependence needs to be measured in order to be inserted in the analysis, therefore the problem reduces to converting the qualitative data into quantitative or, in other wording, to extracting dependence measures from descriptive data. Since this process in not definite, the onus is on the normative expert who acquires and formalises knowledge, to select the appropriate technique for translating qualitative information into useable numbers. The modelling decision may significantly affect the output, depending on the problem formulation and scope. This model uncertainty can be partially investigated by means of sensitivity analysis.

In linguistic terms, dependence can be categorised as non–existent, weak (low), moderate or strong (high). Since associations between cost items are usually captured by correlation coefficients, the type of dependence measure and the three different levels (low, moderate, high) need to be agreed upon. This constitutes another arbitrary yet necessary assumption within the analysis. Technical details of this issue are explained in Chapter 5.

The contribution of individual risk items to the overall risk as well as interactions and associations can be studied using logic tree analysis [FS03]. With respect to that, Fault Trees (FT), Event Trees (ET), Bayesian Networks (BN) and other methods have been developed in order to integrate the concepts of causality and covariation into the system. Each of these approaches has certain limitations regarding the size of projects it can represent in a practical manner.

As stated in Section (4.2), pairwise dependence information does not comprise a full dependence assessment. Yet, this limitation has not been sufficiently emphasised in the literature. Even in case when pairwise dependence is specified, the number of pairs $n(n-1)/2$ can be unsuitably large, for a large number n of events. In practice, risks often form chains of events; in this case, a much smaller number of assessments has to be performed. Moreover, chains are closely related with critical paths for cost and time. This observation can lead to different modelling directions, further discussed in Section (4.7).

Dependence among risks is usually overlooked for several reasons [BS99]:

- Correlations do not appear when single–point estimates are used.
- Dependence is often considered only as a time constraint within an activity schedule.
- Dependence between risks may be present (e.g. in the form of common cause) even if no direct causal relation is obvious.
- A erroneous tautology frequently arises between little or no evidence about dependence, and independence.
- A large number $n(n-1)/2$ of correlations are needed for specifying pairwise dependence.
- The relevant calculus may seem rather unappealing.

It is worth highlighting that the problem of dependence in uncertainty analysis is still under active development [CG04]. The forthcoming section aims to introduce the dependence–aware multivariate problem of cost aggregation.

4.6 Cost aggregation in the multivariate framework

As already discussed in the previous sections, the main subject of the present study is the realistic representation of the aggregate cost, i.e. the probability distribution of the sum of the assessed risks and opportunities. Once the evaluation of individual cost elements has been performed, a set of risks—sometimes referred to as a *portfolio*[9] of risks [BM98]—is defined. The set of risks comprises a random vector $\mathbf{R} = (R_1, R_2, ..., R_n)$, where each of the n individual risks $R_j, j = 1, ..., n$ is a (univariate) random variable. Formula (4.6) shows that it is neither required to

[9]This terminology is primarily used in Financial Risk Analysis.

construct the aggregate loss distribution, nor to consider any information regarding dependence among risks, in order to calculate the expected value of the total loss. When the dispersion of R is also desired, Formula (4.3) in the special case of independent risks yields:

$$\mathrm{Var}[R] = \sum_j \mathrm{Var}[R_j]$$

In the general case, when dependence is additionally considered, the variance of the aggregate distribution is given by [Wan98]:

$$\mathrm{Var}[R] = \sum_j \mathrm{Var}[R_j] + 2\sum_{i<j} cov(R_i, R_j) \tag{4.10}$$

The above formula explains why positive dependencies among risks results to a greater dispersion, hence to higher uncertainty. This will be numerically shown in Chapter 6.

Estimation of cost variance in engineering projects has been the subject of recent studies [WH00]. The variance $\mathrm{Var}[R]$ is very important, since it measures the deviation between average and upper quantile values. Among equivalent risks (in the sense of Equation (2.1)), those events with low probability and high impact should be of more concern [ETKV04]. This recommendation can be explained by the fact that low probability risks contribute more to the variance of the total loss distribution.

The aformensioned effect can be shown as follows: suppose the impact of two risks R_1, R_2 is assessed with point estimates U_1, U_2, and the risks have the same expected value: $\mathrm{E}[R_1] = \mathrm{E}[R_2] = m$, $P[R_1 = U_1] = p_1$, $P[R_2 = U_2] = p_2$. Then for $j = 1, 2$, $\mathrm{E}[R_j] = p_j U_j$, hence $U_j = m/p_j$. Since $\mathrm{Var}[R_j] = \mathrm{E}[R_j^2] - (\mathrm{E}[R_j])^2 = p_j U_j^2 - (p_j U_j)^2 = m^2(1/p_j - 1)$, it holds that:

$$\frac{\mathrm{Var}[R_1]}{\mathrm{Var}[R_2]} = \frac{1/p_1 - 1}{1/p_2 - 1}$$

Hence, it is concluded that if $p_1 < p_2$, then $\mathrm{Var}[R_1] > \mathrm{Var}[R_2]$.

This issue is related to the rare extreme event problem. For instance, a risk may have an extreme impact value, but a very small probability of occurrence. In this case, its expected impact may be equivalent to that of other "normal" risks, but this value is not really informative since the occurrence of this risk can, in fact, invalidate the whole project. Treatment of extreme rare events is still an open question [Tal10], but the usual policy suggests that events evaluated under a certain low likelihood level be altogether ignored.

The correlation effects increase with the number of cost elements [KAE04]. This fact can be explained, not only by the last term of Formula (4.10), but also by the increased associations, hidden beyond the assessed pairwise correlations.

In a multivariate environment, it is possible that some variables have an insignificant influence on the final output. In order to reveal which variables should be modelled stochastically, a sensitivity analysis can be used. Special care should be taken for cost elements and dependencies that strongly affect the total cost; on the contrary, some other model inputs can be considered with an approximate value. According to the "one–way" sensitivity analysis [Rei00] the *swing* of each variable X_k is defined as:

$$S(X_k) = R(X_1, ..., \max(X_k), ..., X_n) - R(X_1, ..., \min(X_k), ..., X_n)$$

where all $X_j, j \neq k$ are fixed at their modal values. Then, the percent variance explained by X_k is defined as:

$$PVE(X_k) = \frac{[S(X_k)]^2}{\sum[S(X_j)]^2}$$

However the validity of this measure relies on the dependence structure of the random vector $(X_1, ..., X_n)$. In general, a shortcoming of most sensitivity analyses is that they ignore possible relationships among the input variables. In particular, the main limitation is the *ceteris paribus* assumption (i.e. that all other things remain the same), when changing a variable [SMJ06, p. 47].

The deviation between the average and the maximum value of the total cost is a measure, representing the additional capital which has to be added to the assumed mean value to yield a safe budget [DDGK05, p. 61]. In financial engineering terms, the Value–at–Risk (VaR) is defined as the upper a–quantile of the total cost distribution F_R [YY05]:

$$\mathrm{VaR}_a(R) = F_R^{-1}(a) \tag{4.11}$$

Over the last years, the Value–at–Risk measure is being gradually replaced by the Expected Shortfall, which is the conditional tail expectation (CTE) of the loss distribution [ADEH99]. This metric was also considered for the present work.

$$CTE_a(X) = \mathrm{E}[X|X > F_R^{-1}(a)] \tag{4.12}$$

Instead of a single number, it is possible to calculate safe intervals for the total anticipated impact. In general, the attempt to obtain upper and lower bounds for multivariate risks usually yields rather wide intervals [GGGR09], [FNH+04] and involves different tools and techniques.

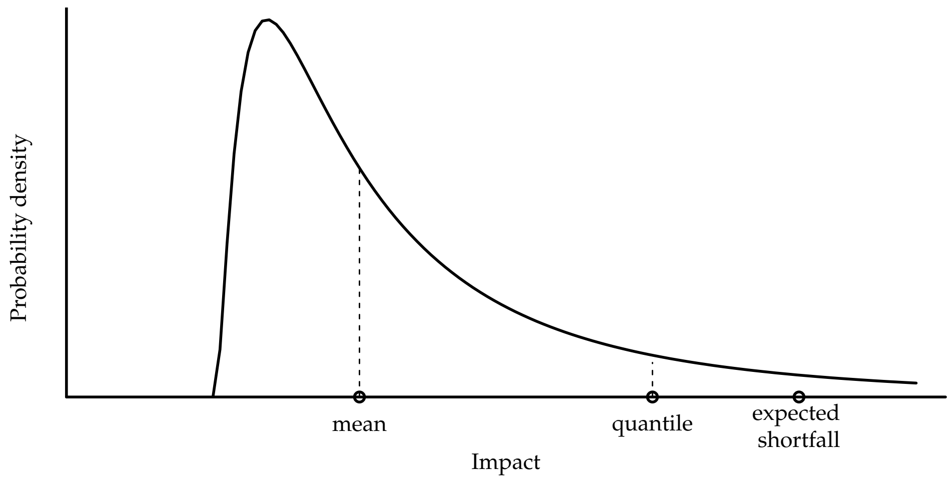

Figure 4.1: Mean value, 95% upper quantile and expected shortfall (mean value of the right tail) of a cost probability density.

4.7 Representations of dependence structures

A multivariate model aims to integrate all available information regarding the individual costs[10] and the inter–variable associations. Therefore, apart from the generic specifications outlined in Section (3.1), there are two basic requirements; the model should [Yan05]:

- Allow arbitrary types for the distributions of individual cost elements.
- Integrate dependencies among cost items.

Individual cost elements have been discussed in Chapter 3. In addition, scalar dependence measures have been outlined in Section (4.4). What remains now, is a strategy to merge the two different objects into a mathematical model. Firstly, it is useful to represent the assessed risks and dependencies on a unified graph. Risks can be depicted as nodes, while risk dependencies as (not necessarily directed) connections between nodes. In general, two main dependence structures among risks can be identified, namely (Figure (4.2)):

[10]The marginal cost distributions are assumed continuous. Most results regarding dependence rely on the continuity assumption for the margins; however, discrete data can also be used [DL05].

1. Structures with no closed cycles. This scheme describes e.g. sequential (or almost sequential) risks.

2. Structures with closed cycles, indicating more complicated interdependencies.

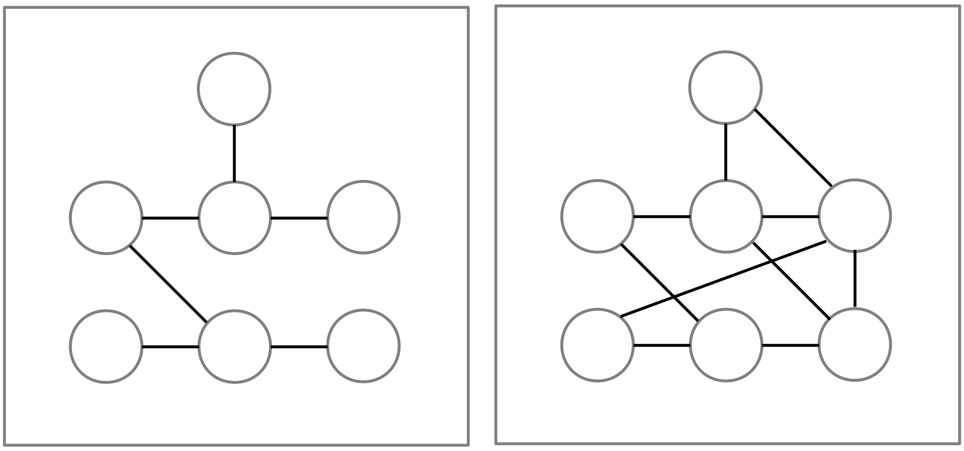

Figure 4.2: Example of dependence structures: acyclic (left) and with closed cycles (right).

In the following, two different approaches aiming to deal with these two cases, are described.

Sequential risks

Activities can have a serial, parallel or overlapping relationship [WH00]. It is often the case that a group of risks can be formed as a sequence $(R_1, ...R_n)$, describing a chain of corresponding activities (workflow) [AM97]. In these cases, the simulation strategy can be formulated as follows: a random value for the first risk R_1 is drawn, and then a value for the second risk R_2 is generated, having the specified underlying dependence between R_1, R_2. The procedure follows the sequence of risks until the last one, and then is repeated by a sufficiently large number of iterations, until the total cost distribution is constructed with acceptable precision. The method of sampling from a series of probability distributions is sometimes referred to as *conditional sampling* [GW04, p. 204].

The above described simulation can be achieved using copula functions [Kel07] (Appendix C). If C is the copula describing the dependence between two variables R_i, R_j, then the conditional distribution of the second variable V, given that the first has been observed $(U = u)$, is:

$$c_u(v) = P[V \leq v | U = u] = \lim_{\mathrm{d}u \to 0} \frac{C(u + \mathrm{d}u, v) - C(u, v)}{\mathrm{d}u} = \frac{\partial C}{\partial u}(u, v)$$

The function $v \to c_u(v)$ exists and it is nondecreasing almost everywhere in $[0, 1]$ (see Appendix C). Therefore, if the value $u = u_0$ has been selected, and an auxiliary uniform variate w_0 is drawn independently from $(0, 1)$, then the value $v_0 = c_u^{-1}(w_0)$ can be calculated, and the pair (u_0, v_0) follows the dependence structure dictated by the copula C. Finally, the pair (x_i, x_j) where $x_i = F_i^{-1}(u_0)$ and $x_j = F_j^{-1}(v_0)$ is sampled from the random pair (R_i, R_j) by probabilistic inversion, where F_i, F_j are the probability functions of the two variables, respectively.

In the following, the described procedure is shown for a particular copula. The Frank copula was selected, since:

- It can capture weak and strong positive and negative dependencies.
- It describes a dependence structure which is visually reasonable.
- The conditional distribution c_u can be easily inverted.

The Frank copula is defined as:

$$C(u, v) = -\frac{1}{\theta} \left(1 + \frac{(e^{-\theta u} - 1)(e^{-\theta v} - 1)}{e^{-\theta} - 1} \right), \theta > 0$$

It readily follows that:

$$c_u(v) = \frac{\partial C}{\partial u}(u, v) = \frac{e^{-\theta u}(e^{-\theta v} - 1)}{(e^{-\theta} - 1) + (e^{-\theta u} - 1)(e^{-\theta v} - 1)}$$

By setting $c_u(v) = w$ and solving by v, it follows that:

$$v = -\frac{1}{\theta} \ln \left(\frac{we^{-\theta} + (1 - w)e^{-\theta u}}{w + (1 - w)e^{-\theta u}} \right) \tag{4.13}$$

The dependence strength is encoded into the parameter θ; the Kendall's tau is related to θ as:

$$\tau = 1 - \frac{4}{\theta} (1 - D(\theta)) \tag{4.14}$$

where D is the Debye function of the first kind, defined as:

$$D(\theta) = \frac{1}{\theta} \int_0^\theta \frac{t}{e^t - 1} \mathrm{d}t \tag{4.15}$$

Table (4.1) shows the five characteristic dependence levels (zero, weak, moderate, strong, perfect) and the corresponding θ values.

Level	Kendall's tau	Parameter θ
independence	0	0
low	0.25	2.37
moderate	0.50	5.74
strong	0.75	14.14
perfect	1	∞

Table 4.1: Theta parameter for the three selected dependence levels, and the two extreme cases (independence and perfect dependence).

When $\tau = 1$ then the copula C reduces to $C(u, v) = uv$ and the calculation is still possible.

The simulation sequence on the risk structure of the left part of Figure (4.2) is shown in Figure (4.3). The simulation, which is not unique, follows the numbered nodes from 1 to 7.

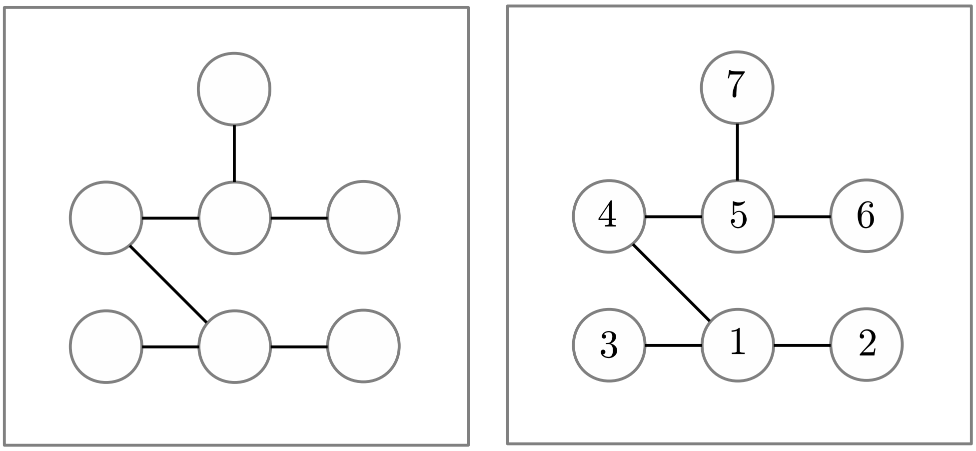

Figure 4.3: Example of stepwise conditional simulation of risks.

Correlation matrix

When the modelled group of risks contains too many dependencies, the correlation matrix approach appears preferable. The $n \times n$ correlation matrix Σ of the random vector $\mathbf{R} = (R_1, ..., R_n)$ contains as entries σ_{ij} the correlation coefficients of the pairs $(R_i, R_j), 1 \leq i, j \leq n$. From the properties of Pearson's correlation coefficient, it is clear that $\sigma_{ij} = \sigma_{ji}$, $\sigma_{jj} = 1$, and $|\sigma_{ij}| \leq 1$. Therefore, the matrix Σ is symmetric, with ones on the main diagonal, and off–diagonal entries within $[-1, 1]$.

Let $\mathbb{M}_n$ denote the class of the $n \times n$ square matrices having the three aforesaid properties (also referred to as *pseudo–correlation* matrices [RM93]). The most important issue concerning the correlation matrix is the feasibility property; a matrix $\Sigma \in \mathbb{M}_n$ is said to be a *feasible* correlation matrix when there exists a random vector $\mathbf{X} = (X_1, ..., X_n)$, having Σ as correlation matrix.

Before mathematically examining the feasibility property, three often arising logical problems of the class $\mathbb{M}_n$ need to be mentioned:

- Incoherence of the matrix entries.
- Measurability of the matrix entries.
- Incompleteness of the matrix.

Incoherence among the matrix entries may appear, since these quantities are inavoidably collected from disparate sources. This fact cannot ensure compatibility among the elements. For instance, if positive dependence exists between (X_1, X_2) and likewise between (X_2, X_3), then (X_1, X_3) cannot be negatively correlated. Measurability of the elements expresses the possibility to precisely quantify dependencies, in terms of correlation coefficients. Measurability, albeit problematic in terms of uncertainty, can prove beneficial when adjusting the matrix to a feasible one. In fact, minor violations of the assumed correlation values are insignificant, given that these values have been quantified in a rather arbitrary manner. Finally, incompleteness results from missing entries. It is worth noting that missing elements are not necessarily attributed to insufficient assessment. The number of required correlations $n(n-1)/2$ can be very large to assess, whereas many of these correlations correspond to dependencies indirectly implied by the assessed ones.

A basic question, regarding the treatment of the missing elements, is whether or not they should be set to zero. In fact, by setting a missing correlation value to zero, specific information on the dependency of concern is being inserted. Since no way to decide upon a missing correlation value really exists, it follows that too many missing entries can result to a large degree of arbitrariness, reflected as significant epistemic uncertainty in the output. In these cases the sequential risk approach—if applicable—is preferable.

A necessary condition for feasibility is that the matrix should be positive semi–definite, in other words, the eigenvalues should be non–negative. The condition can be written as:

$$\mathbf{v}^{\mathrm{T}} \Sigma \mathbf{v} \geq 0$$

for all real non–zero column vectors $\mathbf{v}$ of length n, where $\mathbf{v}^{\mathrm{T}}$ denotes the transpose of $\mathbf{v}$. The above condition is only necessary but not sufficient, nevertheless it imposes a very strong constraint; as n increases, feasibility among elements of the class $\mathbb{M}_n$ is highly unlikely [KC06, p. 52]. This can be intuitively understood by the growing influence of coefficient changes on the roots of the characteristic polynomial (i.e. the eigenvalues), as the degree increases [Jäc02, p. 59].

Any square matrix with non–zero elements can be expressed as the product of a lower triangular matrix L and an upper triangular matrix U (LU decomposition). The following special case is fundamental in the field of random number simulation:

Proposition (Cholesky decomposition): If $\Sigma \in \mathbb{M}_n$ is positive–definite, then there exists a unique lower triangular matrix C, such that:

$$\Sigma = CC^{\mathrm{T}} \tag{4.16}$$

The matrix C is called the *Cholesky* factor of Σ and its elements can be calculated from top to bottom and left to right as:

$$c_{ij} = \frac{\sigma_{ij} - \sum\limits_{s=1}^{j-1} c_{is}c_{js}}{\sqrt{1 - \sum\limits_{s=1}^{j-1} c_{js}^2}}, 1 \leq j \leq i \leq n$$

with the convention that $\sum\limits_{s=1}^{0} (\cdot) = 0$.

The Cholesky factorisation of a matrix Σ can be used for simulating random variables having Σ as their correlation matrix. In order to apply this decomposition, the initial matrix Σ_0 which contains the assessed dependencies needs firstly to be approximated by a positive definite matrix Σ. Therefore, the simulation algorithm, apart from generating samples from the assumed random variates, should be able to [Yan05]:

1. Check whether the matrix of elicited correlations Σ_0 is positive definite.

2. Construct a "close" replacement Σ for $\hat{\Sigma}$, if the condition is not satisfied.

The first is quite simple; it suffices to calculate the matrix eigenvalues and search for negative ones. The second step has seen a number of different developments over the last 30 years (e.g. [Rei00], [Hig02]). Rousseeuw and Molenberghs [RM93] provided an overview of a few solutions; one of them suggests the replacement of negative values in the eigenvalues of Σ with a small positive number. This approach has also been taken by several researchers [BS99], [GH03], [Yan05], [CYC09]. The technique, also referred to as "spectral decomposition" [Jäc02], can be outlined as follows:

1. Analyse the matrix $\Sigma \in \mathbb{M}_n$ as $\Sigma = PEP^{\mathrm{T}}$ (diagonalisation), where E the
2. Replace the negative eigenvalues with a small positive value (say, 0.01) and derive a new vector E^{+}.
3. Calculate the matrix $A = PE^{+}P^{\mathrm{T}}$.
4. If $A = (a_{ij})$, set $d_{jj} = 1/\sqrt{a_{jj}}$, else $d_{ij} = 0$. Set $D = (d_{ij})$.
5. Normalise the matrix A as $\hat{\Sigma} = DAD$.

An even simpler technique suggests the division of the off–diagonal entries by a number greater than one, and check for positive–definiteness. The closest to one is the divisor, the less the original matrix is altered. Other techiques make use of principal component analysis, maximum likelihood estimation, or more sophisticated methods. The hypersphere decomposition method [Jäc02] uses an optimisation technique that additionally allows for assigning different weights to each correlation. In any case, the deviation between the original $\Sigma = (m_{ij})$ and the artificial matrix $\hat{\Sigma} = (m'_{ij})$, can be measured by means of some distance, e.g.:

$$L_{average} = \frac{\sum\limits_{i>j} |m'_{ij} - m_{ij}|}{n(n-1)/2}$$

$$L_{max} = \max_{i>j} |m'_{ij} - m_{ij}|$$

An example of the spectral decomposition technique is shown below:

EXAMPLE 4.2: Suppose the following correlation matrix is given:

$$\Sigma = \begin{pmatrix} 1 & 0.9 & 0.7 \\ 0.9 & 1 & 0.2 \\ 0.7 & 0.2 & 1 \end{pmatrix}$$

The matrix Σ in not positive–definite; its eigenvalues are: 2.24212637, 0.80653659, –0.04866296. Σ is analysed as $\Sigma = PEP^{\mathrm{T}}$, where:

$$P = \begin{pmatrix} 0.6760115 & -0.0457698 & 0.7354683 \\ 0.5658239 & -0.6071484 & -0.5578657 \\ 0.4720718 & 0.7932691 & -0.3845417 \end{pmatrix}$$

$$E = \begin{pmatrix} 2.242126 & 0 & 0 \\ 0 & 0.8065366 & 0 \\ 0 & 0 & -0.04866296 \end{pmatrix}$$

The third negative eigenvalue is replaced by 0.01, and the modified matrix E^{+} is derived:

$$E^{+} = \begin{pmatrix} 2.242126 & 0 & 0 \\ 0 & 0.8065366 & 0 \\ 0 & 0 & 0.01 \end{pmatrix}$$

The matrix A is derived, as $A = PE^{+}P^{\mathrm{T}}$:

$$A = \begin{pmatrix} 1.031732 & 0.8759310 & 0.6834090 \\ 0.875931 & 1.0182567 & 0.2125845 \\ 0.683409 & 0.2125845 & 1.0086746 \end{pmatrix}$$

The matrix A does not have ones in the main diagonal. For that, the auxiliary matrix D is calculated:

$$D = \begin{pmatrix} 0.9845021 & 0 & 0 \\ 0 & 0.9909947 & 0 \\ 0 & 0 & 0.9956907 \end{pmatrix}$$

Finally, the matrix A is normalised by means of the auxiliary matrix D as $\hat{\Sigma} = DAD$:

$$\hat{\Sigma} = \begin{pmatrix} 1 & 0.8545902 & 0.6699183 \\ 0.8545902 & 1 & 0.2097623 \\ 0.6699183 & 0.2097623 & 1 \end{pmatrix}$$

Then, the matrix $\hat{\Sigma}$ is a pseudo–correlation matrix, that can be used in simulation.

4.8 Simulation of correlated variates

In the present section, the procedure of simulating correlated variates for the general case is described. To allow for convenience and clarity, the construction of gaussian copula—the basic tool for performing this simulation—is shown in the bivariate case.

Let Z_1, Z_2 be two independent, normally distributed random variables with zero mean values and unit variances. By applying the affine trasformation:

$$(X_1, X_2) = (\mu_1 + \sigma_{11} Z_1 + \sigma_{12} Z_2, \mu_2 + \sigma_{21} Z_1 + \sigma_{22} Z_2)$$

a pair of normally distributed random variables (X_1, X_2) is obtained, with mean values μ_1, μ_2 and variances $\sigma_1^2 = \sigma_{11}^2 + \sigma_{12}^2$, $\sigma_2^2 = \sigma_{21}^2 + \sigma_{22}^2$, respectively. Moreover, the correlation coefficient between X_1, X_2 is:

$$r = \frac{\sigma_{11}\sigma_{21} + \sigma_{12}\sigma_{22}}{\sigma_1 \sigma_2}$$

The PDF's of Z_1, Z_2 are given by:

$$f_j(z_j) = \frac{1}{\sqrt{2\pi}} \exp\left(-z_j^2/2\right), j = 1, 2$$

For the joint PDF H of Z_1, Z_2, thanks to independence, the following holds:

$$H(z_1, z_2) \mathrm{d}z_1 \mathrm{d}z_2 = f_1(z_1) \mathrm{d}z_1 f_2(z_2) \mathrm{d}z_2 = \frac{1}{2\pi} \exp\left\{-\left(z_1^2 + z_2^2\right)/2\right\} \mathrm{d}z_1 \mathrm{d}z_2$$

The applied transformation can be written in matrix form, as:

$$\begin{pmatrix} x_1 - \mu_1 \\ x_2 - \mu_2 \end{pmatrix} = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$$

If the determinant $\sigma_{11}\sigma_{22} - \sigma_{12}\sigma_{21} \neq 0$, the system can be solved in therms of z_1, z_2:

$$\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix}^{-1} \begin{pmatrix} x_1 - \mu_1 \\ x_2 - \mu_2 \end{pmatrix} = \frac{1}{\sigma_{11}\sigma_{22} - \sigma_{12}\sigma_{21}} \begin{pmatrix} \sigma_{22} & -\sigma_{12} \\ -\sigma_{21} & \sigma_{11} \end{pmatrix} \begin{pmatrix} x_1 - \mu_1 \\ x_2 - \mu_2 \end{pmatrix}$$

Substituting

$$\sigma_1^2 = \sigma_{11}^2 + \sigma_{12}^2, \sigma_2^2 = \sigma_{21}^2 + \sigma_{22}^2, r = \frac{\sigma_{11}\sigma_{21} + \sigma_{12}\sigma_{22}}{\sigma_1 \sigma_2}$$

leads to the relation:

$$z_1^2 + z_2^2 = \frac{1}{1 - r^2} \left\{ \frac{(x_1 - \mu_1)^2}{\sigma_1^2} - \frac{2r(x_1 - \mu_1)(x_2 - \mu_2)}{\sigma_1 \sigma_2} + \frac{(x_2 - \mu_2)^2}{\sigma_2^2} \right\}$$

The Jacobian of the transformation $(x_1, x_2) \to (z_1, z_2)$ is:

$$\frac{\partial(z_1, z_2)}{\partial(x_1, x_2)} = \begin{vmatrix} \partial z_1/\partial x_1 & \partial z_1/\partial x_2 \\ \partial z_2/\partial x_2 & \partial z_1/\partial x_1 \end{vmatrix} = \begin{vmatrix} \sigma_{11}/r' & -\sigma_{12}/r' \\ -\sigma_{21}/r' & \sigma_{22}/r' \end{vmatrix} = \frac{1}{r'} = \frac{1}{\sigma_1 \sigma_2 \sqrt{1 - r^2}}$$

where

$$r' = \sigma_{11}\sigma_{22} - \sigma_{12}\sigma_{21} = \sigma_1\sigma_2\sqrt{1-r^2}$$

Thus

$$\mathrm{d}z_1\mathrm{d}z_2 = \frac{\mathrm{d}x_1\mathrm{d}x_2}{\sigma_1\sigma_2\sqrt{1-r^2}}$$

Therefore, the joint distribution of X_1, X_2 is derived as:

$$f(x_1, x_2; r) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-r^2}} \exp\left\{-z^2/2(1-r^2)\right\}$$

where:

$$z^2 = \frac{(x_1-\mu_1)^2}{\sigma_1^2} - \frac{2r(x_1-\mu_1)(x_2-\mu_2)}{\sigma_1\sigma_2} + \frac{(x_2-\mu_2)^2}{\sigma_2^2}$$

In the general case of $n > 2$ random variables with mean values $\boldsymbol{\mu} \in \mathbb{R}^n$ and correlation matrix Σ, the multivariate normal distribution can be similarly derived as a PDF:

$$f_{\mathbf{X}}(x_1, ..., x_n; \Sigma) = \frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}} \exp\left(-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^{\mathrm{T}}\Sigma^{-1}(\mathbf{x}-\boldsymbol{\mu})\right) \tag{4.17}$$

When two or more random variables are jointly normal, then it can be readily shown that the marginal distributions are also normally distributed. This assumption is very restrictive in the general case. However, using the multivariate version of the above formula, it is possible to construct the gaussian copula, based on Sklar's theorem (Appendix C). The latter can be used to represent a dependence structure that is fully determined by a correlation matrix, but the marginal distributions can be of any desired density.

Suppose the random vector $\mathbf{R} = (R_1, ..., R_n)$ with correlation matrix Σ. Denote by F_j the CDF of the variable $R_j, j = 1, ..., n$. Moreover, suppose the dependence structure is described by a gaussian copula:

$$C(u_1, ...u_n) = \Phi_\Sigma\left(\Phi^{-1}(u_1), ..., \Phi^{-1}(u_n)\right)$$

where Φ is the CDF of the univariate standard normal distribution,

$$\Phi(x) = \frac{1}{2\pi}\int_{-\infty}^{x} \exp(-t^2/2)\mathrm{d}t$$

and Φ_Σ is the CDF of the distribution expressed in Formula (4.17).

Pearson's linear correlation coefficients are directly related to Spearman's rho and Kendall's tau rank correlations as follows [LMS03]:

$$r_{ij} = \sin\left(\frac{\pi}{2}\tau_{ij}\right) \tag{4.18}$$

$$r_{ij} = 2\sin\left(\frac{\pi}{6}\rho_{ij}\right) \tag{4.19}$$

The simulation procedure, usually referred to as *Normal–to–Anything* (NORTA) can be formulated as follows [Yan11], [GH03]:

1. A matrix Σ_0 containing the assessed rank correlations is formed.
2. An initial correlation matrix Σ_1 is calculated according to Formula (4.18).
3. The matrix Σ_1 is approximated by a positive definite matrix Σ, as described in the previous section.
4. The Cholesky factorisation of Σ is performed [Wan98], yielding a lower triangular matrix C.
5. A vector $\mathbf{w} = (w_1, ..., w_n)$ of independent (uncorrelated) standard uniform variates is drawn.
6. A vector $\mathbf{z} = (z_1, ..., z_n) = (\Phi^{-1}(w_1), ..., \Phi^{-1}(w_n))$ of independent (uncorrelated) standard normal variates is calculated.
7. The vector $\mathbf{z}$ is tranformed to $\mathbf{t} = C\mathbf{z}$, which is multinormally distributed with correlation matrix Σ.
8. The vector $\mathbf{u} = \Phi(\mathbf{t})$ is calculated.
9. The numbers $u_j, j = 1, ..., n$ are transformed to costs by inversion $x_j = F_j^{-1}(u_j)$.
10. Steps 5–9 are iterated to generate a sufficient number of samples.

Clemen and Reilly [CR99] noted that the above method is essentially a copula–based technique. The multivariate normal copula is constructed by means of the multivariate normal distribution, but this does not impose any limitation with regard to the choice of the marginal distributions. After the above simulation, the statistics of the total cost can be calculated. The method has been further refined by other researchers (e.g. [Sta05]).

A different approach, dating back to the work of Iman and Conover [IC82] suggests applying permutations on the sampled variates in order to approximately achieve the target correlations. This distribution–free method was further explored in several studies [CP04], [Cha06], [VN09], [Vor10] and enhanced with variance reduction techniques, such as Latin Hypercube Sampling (LHS). The problem of evaluating the uncertainty pertaining to the selected method for generating correlated variates has also been investigated [Haa99], [KKV07], [DL09], [TSB11a], [TSB11b].

The following example showcases the above described algorithm:

EXAMPLE 4.3: Suppose three beta densities with the correlation matrix Σ of EXAMPLE (4.2) and parameters given at Table (4.2). The Cholesky decomposition of the correlation matrix is:

$$C = \begin{pmatrix} 1 & 0 & 0 \\ 0.8545902 & 0.5193030 & 0 \\ 0.6699183 & -0.6985194 & 0.2515554 \end{pmatrix}$$

Next, a vector of length 3 of uncorrelated uniform variates is randomly drawn:

$$\mathbf{w} = (0.5458081, 0.9797237, 0.5118450)$$

The vector $\mathbf{z} = (z_1, z_2, z_3) = (\Phi^{-1}(w_1), \Phi^{-1}(w_2), \Phi^{-1}(w_3))$ of independent (uncorrelated) standard normal variates is calculated:

$$\mathbf{w} = (0.1150773, 2.0480755, 0.0296953)$$

The vector $\mathbf{z}$ is tranformed to $\mathbf{t} = C\mathbf{z}$:

$$\mathbf{t} = \begin{pmatrix} 1 & 0 & 0 \\ 0.8545902 & 0.5193030 & 0 \\ 0.6699183 & -0.6985194 & 0.2515554 \end{pmatrix} \begin{pmatrix} 0.1150773 \\ 2.0480755 \\ 0.0296953 \end{pmatrix} = \begin{pmatrix} 0.115077 \\ 1.161915 \\ -1.346058 \end{pmatrix}$$

The vector $\mathbf{u} = \Phi(\mathbf{t})$ is calculated:

$$\mathbf{u} = \Phi(0.115077, 1.161915, -1.346058) = (0.5458081, 0.8773651, 0.0891418)$$

Finally, the vector $\mathbf{u}$ is transformed by probabilistic inversion of the beta CDF's:

$$(5.983221, 6.215859, 5.237399)$$

Distribution	L	U	α	β
1	0	10	3.34	2.56
2	2	8	4.00	4.00
3	4	12	6.28	16.84

Table 4.2: The three beta densities of EXAMPLE (4.3), with their parameters.

4.9 Concluding remarks

From the exposed theoretical investigation, the following tools were adopted for the case study in Chapters 5 and 6:

- The generalised beta distribution for the probabilistic representation of individual risks.
- The Kendall's tau for measuring the (not necessarily linear) association between pairs of risks.
- The gaussian copula for modelling the dependence structure of the entire set of risks.

Chapter 5

Case study: the Brenner Base Tunnel

5.1 Introduction to the BBT project

The present section intends to provide a brief description of the Brenner Base Tunnel (BBT), following the articles [Ber11a], [QBFM10] and books [Ber08], [Ber11b].

The Brenner Pass, one of the main passes of the Alps, is situated along the border between Italy and Austria. It has been an important mountain passage since the Roman era, and still is the most significant North–South connection in the EU. Right after the Second World War, the possibility for a tunnel construction in the area was conceived. The first technical feasibility study dates back to 1989. As of 2012, the tunnel is under construction upon design principles for a service lifetime of 200 years.

The BBT is a flat trajectory railway tunnel running between Innsbruck, Austria and Fortezza/Franzenfeste, Italy. Combined with an already existing underground bypass in Innsbruck, it will be the world's longest underground railway, reaching a total length of about 64 km. The BBT is a critical part of the 2,200 km high–speed railway axis Berlin – Munich – Verona – Bologna – Palermo. This route is part of the Trans–European Networks EU program, which aims to contribute to the development of European market by improving the economic and social cohesion within the Community and the standardisation of transport system.

The tunnel is nearly horizontal with a longitudinal gradient 5.0‰ to 6.7‰. It consists of two parallel tubes, and is designed for a maximum speed of 250 km/h. The crown height is 795 m above sea level, the net cross–section of the main tubes about 43 m^2, the minimum cross–section of the exploratory tunnel about 26 m^2 and the clearance of the cross–cuts 300 m. An exploratory tunnel runs between the two tubes and 12 m deeper than the main tunnel. This exploratory tunnel will be built before the construction of the main tubes, primarily for exploring the rock mass; moreover, it will serve as a drainage tunnel or as a service tunnel, if need be. The whole project was authorised by the governments of Austria and Italy in 2009.

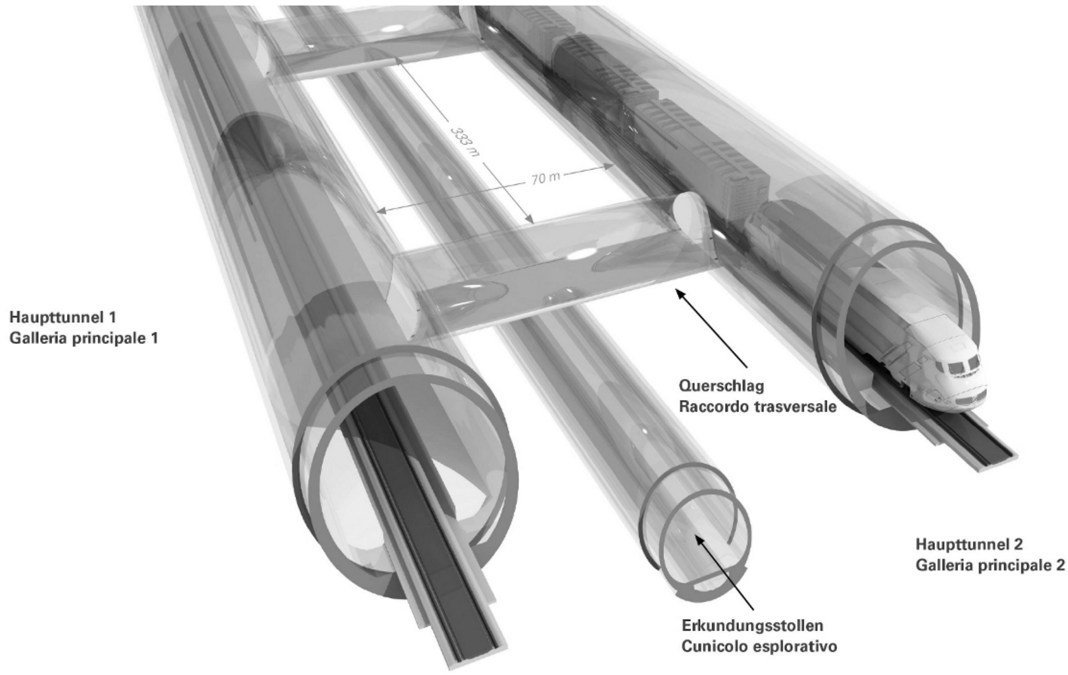

Figure 5.1: The two main tubes and the exploratory tube of BBT (source: The Brenner Base Tunnel website [bbt]).

The preliminary project was developed in phase I, from 1999 to 2002. In phase II, from 2003 to 2008, technical and environmental compatibility planning was completed, and a large number of inspection bores were opened. From 2010 to 2011, further financing for phase III (building phase) was approved. With regard to the current status (2012), the preparatory work and part of the exploratory tunnel and the access tunnel have been completed. The beginning of construction works of the main lots is scheduled for 2016 and the completion of the project is planned for 2025.

The tunnel is expected to reduce transportation times, to improve traffic flow organisation and to safeguard the Alpine environment by reducing CO_2 emissions. The whole BBT project is characterised by an enormous technical complexity, requiring interdisciplinary expertise, involvement of several parts and accomplishment of a broad variety of tasks within a long–term procedure. The enterprise life–cycle

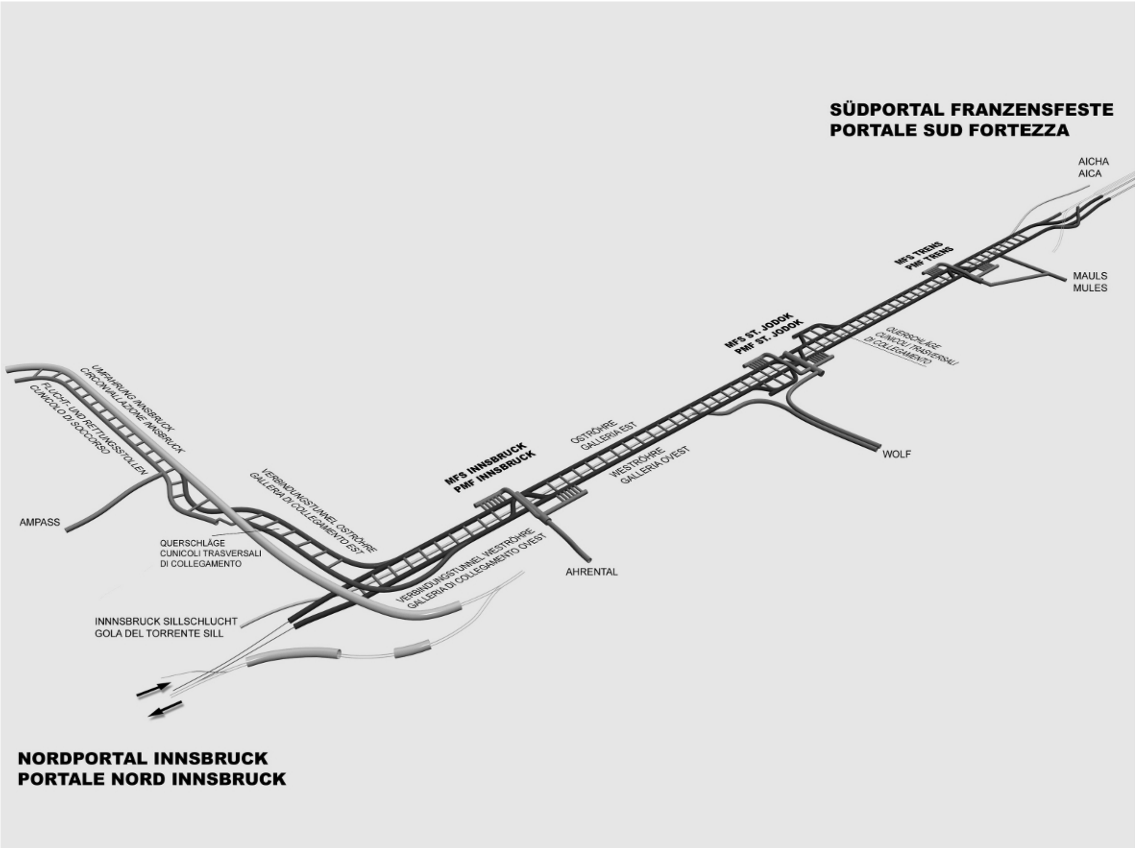

Figure 5.2: Overview of the BBT project (source: The Brenner Base Tunnel website [bbt]).

(conception, definition, execution) involves numerous operations from the early design phase until completion; apart from the technical requirements, decision–makers have to deal with public procurement and managerial issues, economic challenges, and governmental and European policies. These particular demanding conditions call for advanced and refined processes for managing knowledge, resources, activities and risks.

5.2 Sources of information and uncertainty

The costruction schedule prepared for the BBT project has the form of a workload chart placed upon clearly visible time phases. The graph, shown in Figure (5.3), also internally known as "Bergmeister Plan" [Alf12], covers the entire range of construction activities. The diagram provides an excellent basis for risk analysis: every activity can be directly pinpointed and associated with its time and location attributes. From temporal, spatial and causal properties of the planned activities, most of the corresponding risks can be identified and placed on the diagram. This plan also allows for classifying risks into groups, which play an important role for risk management, as well as for the simulation process.

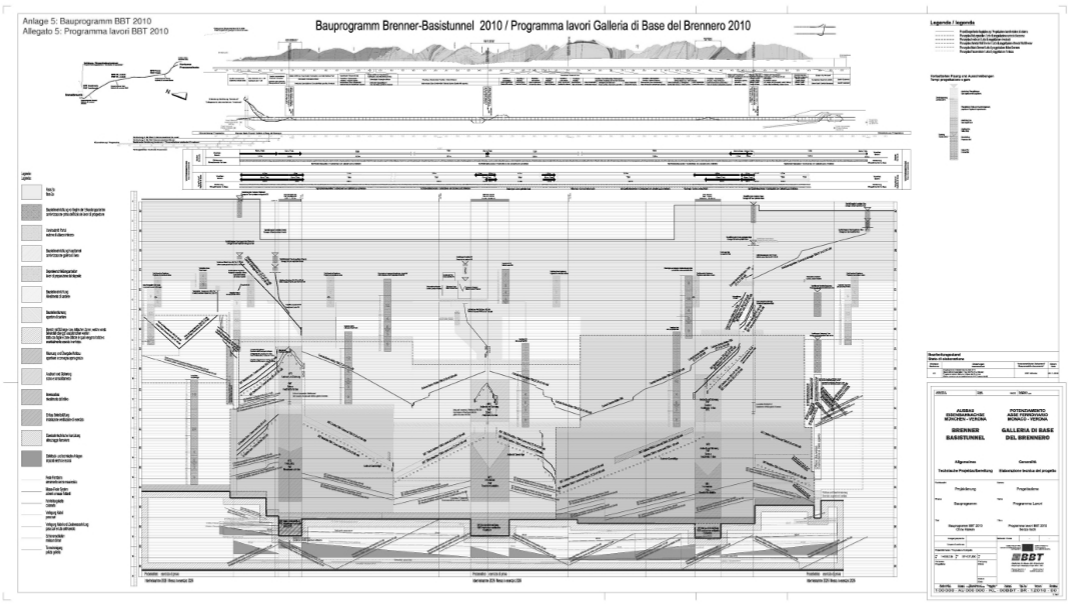

Figure 5.3: The Bergmeister plan (source: The Brenner Base Tunnel website [bbt]).

Since most of the risks are associated with planned activities, their assessment is based on work path, time and location; this mostly applies to technical risks. Information from the exploratory tunnels and other acquired geological data should be managed and documented with particular care. The resulting reports and summaries, apart from their technical significance with regard to construction, they can provide a sound background for risk assessment. Non–technical risks can be derived from fiscal analyses, legal reports, Value for Money and Cost–Benefit analyses, market testing and project–specific knowledge. In the case of BBT, the corresponding documents provide an invaluable source of information and deeper insight into the project.

As discussed in the previous chapters, the present study is particulary interested in the risk term R of the total cost, as expressed by Formula (2.2). The base cost B encompasses the expenses for all realisation phases, planned activities, provisioned material procurement, human work and market conditions. In particular, in tunnelling projects, B is derived from elementary costs, i.e. costs for excavation classes, site equipment, lining, ventilation, etc. [PS06], [PSP07]. In the case of BBT, the base cost has come as the result of a minute and exhaustive spreadsheet calculation.

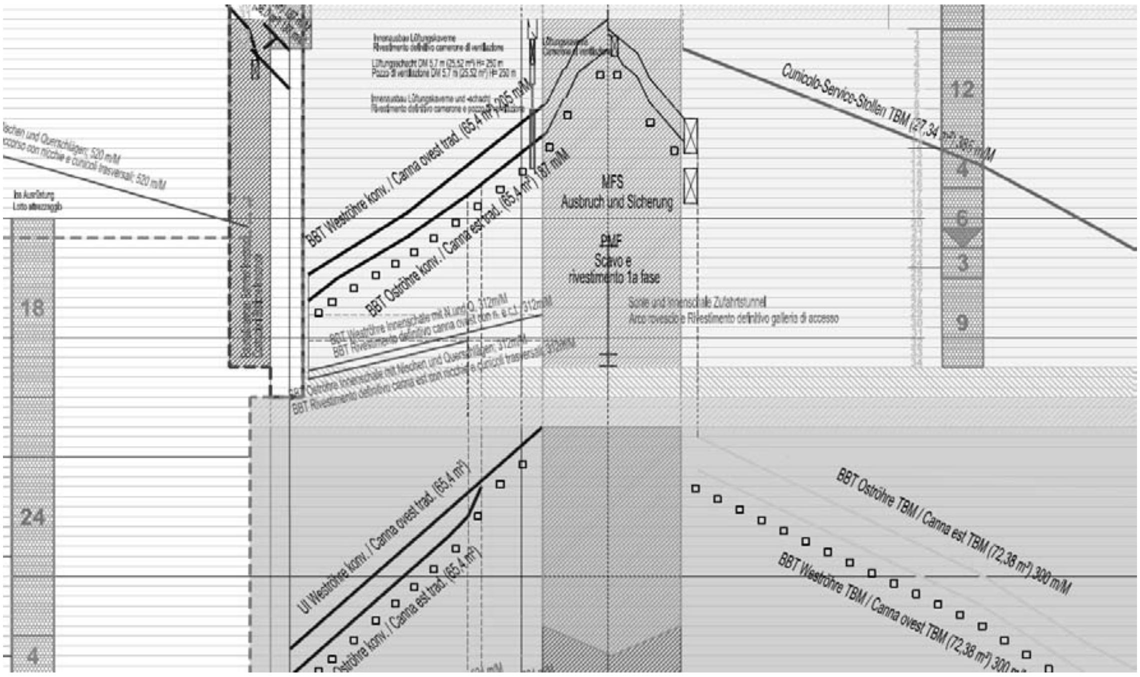

Figure 5.4: The Bergmeister plan – detail (source: The Brenner Base Tunnel website [bbt]).

Due to significant variability and large epistemic uncertainties, the risk term R is treated as a random variable. The PDF of this variable is constructed by individual cost items and interdependencies. The uncertainty in the individual cost elements is based upon expert beliefs, and modelled as described in Chapter 3. Apart from subjective opinions based on real data, common sense can be used to deal with large information gaps. However, it is worth noting that common sense could both buttress and hinder knowledge discovery, and should be used with proper care.

Since uncertain quantities are not pre–formed numbers waiting to be elicited, personal beliefs regarding these quantities are being formed within the elicitation process; therefore they depend on the linguistic context [OBD+06]. Hence, the elicitation process, which is formulated to address uncertainty can itself, to some degree, generate additional uncertainty.

Finally, uncertainty is generated as the smaller parts are assembled and included into an integrated computational model. For instance, the selection of the multivariate model plays an important role [KKV07]. Moreover, when the individual (marginal) distributions are assessed separately from correlations, the transfer of information from one margin to another is not possible [Seg06]. False assumptions in the model can impair the validity of collected data, since conceptual errors are the most devastating ones [BC00]. In any case, there is no such thing as perfect uncertainty modelling since uncertainty is, to a great extent, imperfect knowledge. The need to distinguish among variability, imperfect information and conceptual flaws highlights the importance of uncertainty classification, discussed in Section (2.3).

5.3 Risk classification in BBT

A qualitative risk assessment usually aims to cover the following issues: (1) description of the risk, (2) project's stage when it may occur, (3) elements that could be affected, (4) influence factors, (5) dependencies with other risks, (6) likelihood, and (7) impact of the risk [SMJ06, p. 39]. This was exactly what was followed in BBT: the project's risk analysis, as of 2012, is mainly based upon the work conducted in 2008, documented by Alfreider [Alf09], and made internally available. The risks were derived from workshops and interviews with BBT engineers and experts, as well as with external consultants.

The following classification was applied in the risk analysis of BBT:

- Technical risks: risks related to subsoil conditions, construction and excavation, execution of any type of engineering works, logistics, etc.
- Technical–administrative risks: contractual risks, planning variations, normative and legal modifications, risks related to administrative procedures, lack of resources, etc.
- Authorisation risks: risks linked with unsuccessful agreements with interfered bodies, delays and unforeseen issues in the authorisation procedures, additional measures, difficulties in the process of expropriation, etc.

The above classification is very useful for risk management purposes, but only indicative in the present context; also, it does not account for risk grouping, which is described later in Section (5.6).

5.4 Assessment of individual cost elements

For every identified risk, the following attributes were determined:

1. The risk identity, a codename representing the risk within the analysis.
2. The group, to which the risk belongs.

3. The baseline cost (evaluation basis), a concept discussed in Section (3.1)—where applicable.
4. The level of confidence in assessment, defined in Section (3.3).
5. The probability of occurrence, if applicable.
6. The lower, most likely and upper risk impact, either as percentages of the evaluation basis, or as absolute economic values. All these quantities hereafter are measured in million EUR.

At this point, the following three assumptions should be clarified:

Minimum values

For all the 89 assessed risks, the minimum value was set to zero. This was due to limited information at the assumed phase, and does not constitute a modelling choice nor a computational constraint. The quantity referred to as "risk importance" in [Alf09] was set as modal value, while the maximum value was adopted, as assessed in the same analysis. These three quantities comprise the triad (L, M, U) used for the construction of individual cost densities.

Probability of occurrence

At this early point, the concept of "probability of occurrence" was not considered. Although some of the assessed risks have in fact the possibility of not appearing, the corresponding likelihood was ignored. This assumption is apparently on the safe side, and will be revised as soon as newer information appears. In general, the assessment of probabilities can be problematic, especially in the presence of large uncertainties and with untrained experts. Issues that frequently appear, for instance, are (1) the "unpacking principle", where more detailed descriptions of an event increase its judged probability, and (2) the inflation of probability sums when the number of sub–events comprising an event increases [OBD+06]. These caveats call for a careful and methodological probability assessment process.

Confidence in assessment

The lack of confidence in assessment, the fourth parameter used to build probability densities, was set to the high value of 0.42 for all risks. At the point of writing, there were no particular assessments of this quantity; this is planned for future steps. Nevertheless, a sensitivity analysis carried out and presented in Chapter 6 aims to investigate the influence of this parameter in the final output. With regard to the elicitation process, several formats have been proposed. Hahn [Hah08] proposes the simple formulation "*How certain are you that the mode is truly M?*". If the question is of the type "*How confident are you about your/this assessment?*" it focuses on epistemic uncertainty, whereas a form '*'How certain/likely is your/this assessment?*" leans towards the aleatory side [OBD+06]. An interesting technique based on the theory of "potential surprise"[1] proposes that the probability of an event is inversely related to the surprise that one would experience at the event's occurrence [Sha49]. When the aforesaid probability is the second–order probability describing the confidence in assessment, this technique can also serve for the assessment of confidence.

Table (5.1) summarises the collected data to be utilised in the further computation. T stands for technical, V for technical–administrative, and A for authorisation risks. The numbering of risks (labelled as "name") is adopted from the BBT official documentation [Alf09].

name	**minimum**	**mode**	**maximum**	c
T–1	0	1.5	3.0	0.42
T–2	0	1.0	5.0	0.42
T–3	0	1.2	2.0	0.42
T–4	0	3.0	5.0	0.42
T–5	0	4.0	20.0	0.42
T–6	0	0.4	2.0	0.42
T–7	0	0.8	4.0	0.42
T–56	0	5.0	10.0	0.42
T–57	0	10.0	20.0	0.42
T–8	0	28.0	40.0	0.42
T–9	0	35.0	50.0	0.42
T–10	0	24.0	30.0	0.42
T–11	0	40.0	50.0	0.42
T–12	0	0.8	1.0	0.42
T–13	0	1.6	2.0	0.42
T–14	0	0.4	1.0	0.42

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[1] Found also as "principle of least astonishment" in other scientific fields [CMD02].

Table 5.1 – *...continued from previous page*

name	**minimum**	**mode**	**maximum**	c
T–53	0	0.8	2.0	0.42
T–15	0	20.0	50.0	0.42
T–51	0	2.9	7.3	0.42
T–52	0	4.3	10.7	0.42
T–58	0	16.0	20.0	0.42
T–59	0	32.0	40.0	0.42
T–16	0	21.0	30.0	0.42
T–17	0	14.0	20.0	0.42
T–18	0	9.0	30.0	0.42
T–19c	-40.0	-8.0	0	0.42
T–20	0	24.0	30.0	0.42
T–21	0	32.0	40.0	0.42
T–60	0	1.5	5.0	0.42
T–61	0	8.0	20.0	0.42
T–22	0	3.0	10.0	0.42
T–23	0	6.0	20.0	0.42
T–24	0	6.0	20.0	0.42
T–25c	-49.0	-4.9	0	0.42
T–26	0	24.0	80.0	0.42
T–27	0	0.3	1.0	0.42
T–28	0	2.5	5.0	0.42
T–29	0	3.0	10.0	0.42
T–30	0	4.0	10.0	0.42
T–31	0	0.6	2.0	0.42
T–32	0	3.0	10.0	0.42
T–33c	-6.1	-1.2	0	0.42
T–54	0	3.7	12.3	0.42
T–34	0	0.6	2.0	0.42
T–35	0	1.5	5.0	0.42
T–36	0	3.0	10.0	0.42
T–37	0	1.5	5.0	0.42
T–38	0	3.0	10.0	0.42
T–39	0	8.0	20.0	0.42
T–55	0	16.0	40.0	0.42

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Table 5.1 – *...continued from previous page*

name	**minimum**	**mode**	**maximum**	c
T–40	0	9.0	30.0	0.42
T–41	0	4.9	49.0	0.42
T–42	0	1.8	18.4	0.42
T–43	0	4.2	42.4	0.42
T–44	0	7.2	18.0	0.42
T–45c	-49.0	-4.9	0	0.42
T–46c	-15.2	-7.6	0	0.42
T–47	0	15.5	51.8	0.42
T–48	0	3.3	11.0	0.42
T–50	0	9.6	24.0	0.42
V–1	0	0.0	0.0	NA
V–2	0	28.1	56.3	0.42
V–3	0	0.6	5.7	0.42
V–4	0	0.6	1.2	0.42
V–6	0	7.5	37.7	0.42
V–7	0	8.0	16.0	0.42
V–8	0	3.2	10.5	0.42
V–9	0	27.0	54.0	0.42
V–10c	-37.7	-7.5	0	0.42
V–11	0	6.4	16.0	0.42
V–12	0	15.1	37.7	0.42
V–13	0	3.2	8.0	0.42
V–14	0	3.8	37.7	0.42
V–15	0	8.0	40.0	0.42
V–16c	-37.7	-4.0	0	0.42
V–17c	-8.0	-2.0	0	0.42
V–18	0	0.0	0.0	NA
V–19	0	0.3	1.7	0.42
V–20	0	1.0	5.1	0.42
V–21	0	0.3	1.7	0.42
V–28	0	15.1	37.7	0.42
V–29	0	4.4	11.0	0.42
V–30	0	5.4	27.0	0.42
V–31	0	6.4	12.8	0.42

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Table 5.1 – *…continued from previous page*

name	**minimum**	**mode**	**maximum**	c
A–1a	0	1.7	8.5	0.42
A–1b	0	6.6	33.0	0.42
A–2	0	10.0	20.0	0.42
A–3	0	20.0	100.0	0.42
A–4	0	8.0	20.0	0.42

Table 5.1: List of the risks and opportunities with assessed minimum, mode, maximum and lack of confidence values.

In the following section, the two most widely used methods to present a collection of risks, along with a new proposed one, are exposed.

5.5 Risks schemes and presentation

After the identification, assessment and quantification process, risks should be visually represented to provide a risk overview. This needs to be done in a clear and comprehensive manner, in order to deliver the basic ideas quickly. Moreover, the picture has to be representative, so that it reflects the true situation.

The first task of the analyst is to define the risk importance. As discussed in Section (2.1), risks are evaluated by means of Formula (2.1)[2]. This methodology applies to risks, which are expressed as single–point assessments, and possess a likelihood of occurrence. In the preceding section, the role of the probability concept in the current state of the analysis was explained. Moreover, in Section (3.1) it was described how the individual assessments go beyond single–point quantities. Still, it is possible to follow the techniques built upon the "traditional" treatment of risks, according to Formula (2.1). In particular, when a risk is assessed as $E = P \cdot U$, three facts regarding the risk are disclosed:

1. The average/expected risk value is E.
2. The maximum value of the risk is U.
3. The maximum value occurs with probability P and zero value occurs with probability $1 - P$.

[2]Another way to compare risks is by their "risk factor", defined as $RF = P + C - P \cdot C$ where P the probability and C the risk consequence, scaled in $(0, 1)$ [CGRW04].

When the risk is assessed as a probability beta density with parameters L, M, U, c, similar information is at hand:

1. The modal risk value is M.
2. The maximum value of the risk is U.
3. Any desired probabilities are expressed by the risk's PDF.

In the risk analysis conducted for BBT [Alf09], the "probability" of a risk was defined as the ratio E/U. It can be shown that this assumption is indeed consistent with the transition from single–point assessments to probability densities. Firstly, it is noted that the quantity $P/(1-P)$ expresses the likelihood of occurrence, devided by the likelihood of non–occurrence. Then:

$$\frac{P}{1-P} = \begin{cases} 0 & , P \to 0 \\ 1 & , P = 1/2 \\ \infty & , P \to 1 \end{cases}$$

The three cases $P \to 0$, $P = 1/2$, $P \to 1$ express the situations of positive (right–hand) skewness, symmetry, and negative (left–hand) skewness. The same conditions can be expressed in terms of the PDF's parameters as follows:

$$\frac{M-L}{U-M} = \begin{cases} 0 & , M \to L \\ 1 & , M = (L+U)/2 \\ \infty & , M \to U \end{cases}$$

By setting:

$$\frac{P}{1-P} = \frac{M-L}{U-M}$$

and solving for P, it yields that:

$$P = \frac{M-L}{U-L}$$

Finally, for $L = 0$ (which is the case for the current assessments):

$$P = \frac{M}{U} \approx \frac{E}{U}$$

The above approximation relies on the assumption that the modal and the average values are acceptably close. The absolute difference between these two quantites, by means of Equations (3.10) and (3.11) is:

$$|E-M| = (U-L)\left|\frac{\alpha}{\alpha+\beta} - \frac{\alpha-1}{\alpha+\beta-2}\right| = \frac{(U-L)|\beta-\alpha|}{(\alpha+\beta)(\alpha+\beta-2)}$$

Using Formula (3.22) to eliminate β, it yields that:

$$|E-M| = \frac{(U-L)|1-2m|}{(\alpha-1)/m+2}$$

Substituting α by the expression of Formula (3.17) and noting that $(U-L)|1-2m| = |U+L-2M|$, it is derived that:

$$|E-M| = \frac{|U+L-2M|}{(1-3c^2)+(1-c^2+\sqrt{D})/m}$$

By Inequality (3.26) it follows that:

$$|E-M| \leq \frac{|U+L-2M|}{2}$$

The deviation between the "pseudo–probabilities" $P^* = E/U$ and $P = M/U$ is:

$$|P^*-P| \leq \left|\frac{E}{U} - \frac{M}{U}\right| \leq \frac{|U+L-2M|}{2U} < 0.50$$

The above formula produces rather conservative bounds. For the 89 assessed risks, in 70 cases the real error $|P^*-P|$ is lower than 10%, while the maximum error is 37% in some very skewed assessments (i.e. T–25c). It is concluded that the choice of defining the ratio between the average and the downside risk value, is justified, given that the assumption does not serve computational but only visualisation needs. Also, the emerging large deviations highlight the importance of distinguishing between the most likely and the average cost during the assessment process.

The remainder section aims to provide an overview of the common methods used to depict the whole set of risks, so as to give a quick picture of the risk assessment to the analysts [Ker09, p. 744], [PS06]. The modal and average risk value are both denoted by M.

The Expected impact – Probability graph

This graph shows the expected impact of the risks and their probability of occurrence (or M/U) as x–y coordinates (5.5).

Suppose a risk at the point $Q = (M, P)$. When moving to the right on the horizontal line $y = P$ one finds risks with the same probability of occurrence $P' = P$, and greater modal value $M' > M$. If $Q' = (M', P')$ is one such new point, then its maximum impact is $U' = M'/P' > M/P = U$. Therefore, the risk $Q' = (M', P')$

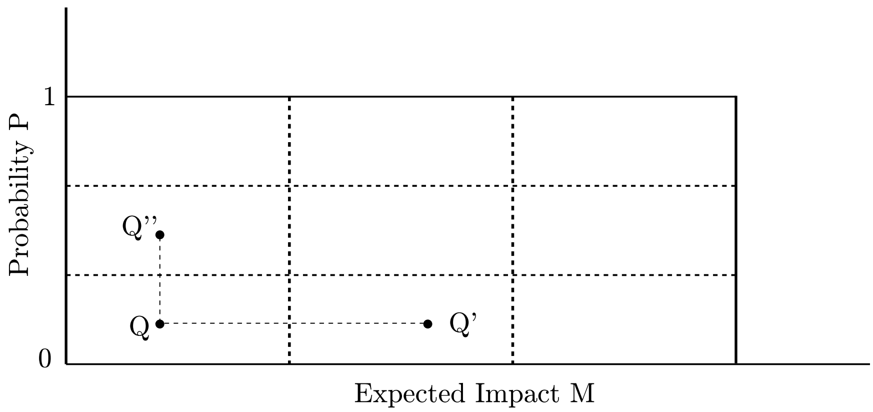

Figure 5.5: The Expected impact – Probability graph.

is more unfavourable (as discussed in Section (4.6)). Suppose now a risk $Q'' = (M'', P'')$ higher than $Q = (M, P)$ on the vertical line $x = M$. Then the two risks Q, Q'' possess the same expected impact[3] $M'' = M$, but $P'' > P$. Therefore $U'' = M''/P'' < M/P = U$, so the risk $Q'' = (M'', P'')$ is less unfavourable.

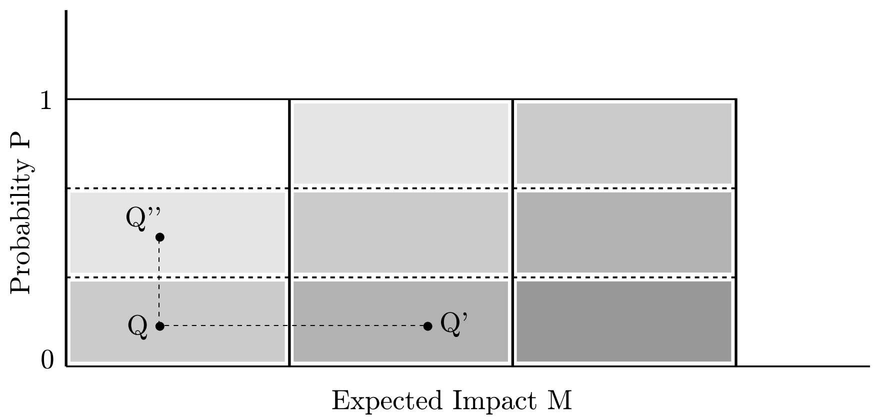

Figure 5.6: The Expected impact – Probability graph divided into areas, according to the risk's severity (maximum impact).

This scheme is easy to produce, yet it may give a misleading picture of the risk importance, the same way that expected value decision–making can be misleading for extreme rare events [Ave11]. Moreover, it offers a way to directly compare any two risks in terms of expected impact (x–coordinate), but not in terms of maximum impact.

[3] With due reservation for small deviations between modal and average values, as explained earlier which, in any case, do not affect the essence of the argument.

The Maximum impact – Probability graph

This graph shows the maximum possible impact U (which is always larger than the expected, and expressed in the same monetary unit) of the risks and their probability of occurrence (or M/U) as x–y coordinates (Figure (5.7)):

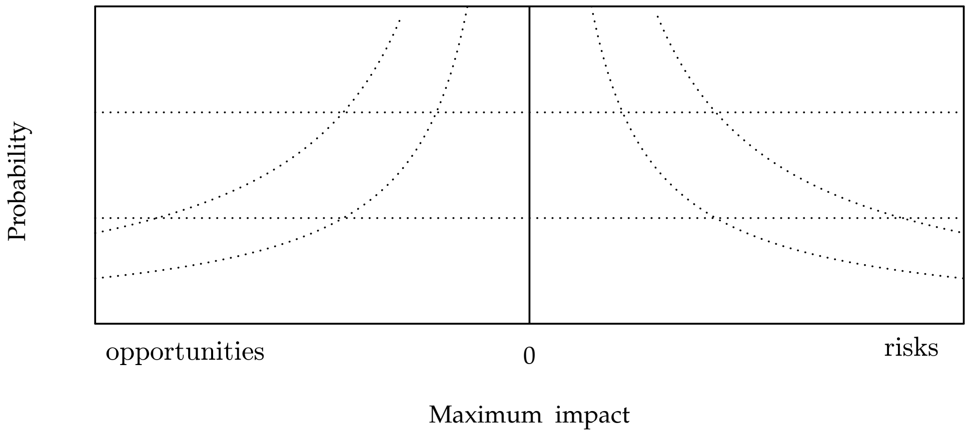

Figure 5.7: The Maximum impact – Probability graph divided into areas, according to the risk's expected impact.

Suppose a risk at the point $Q = (U, P)$. Then $x = U$ and $y = P$, therefore $y \cdot x = P \cdot U = M$. Therefore, all events with equal expected value M lie on the same hyperbola $y = M/x$. When moving to the right of one such hyperbola, one meets risks with greater maximum value, hence more unfavourable. This graph makes it easier to directly compare any two risks in terms of maximum impact (x–coordinate) but less obvious in terms of expected impact.

The relation between maximum impact and probability can also be represented in logarithmic coordinates [Tod06, p. 61]. The basic formula $M = P \cdot U$ can be written also as:

$$\log M = \log P + \log U$$

Therefore, equivalent risks belong on the same straight line. The latter expression carries the limitations that exist in formula $M = P \cdot U$; linearity in the log space can be affected e.g. by risk aversion factors.

The Inverse Probability graph — Maximum impact

This graph shows the inverse probability (rarity) in the x–axis and the maximum impact in the y–axis (Figure (5.8)).

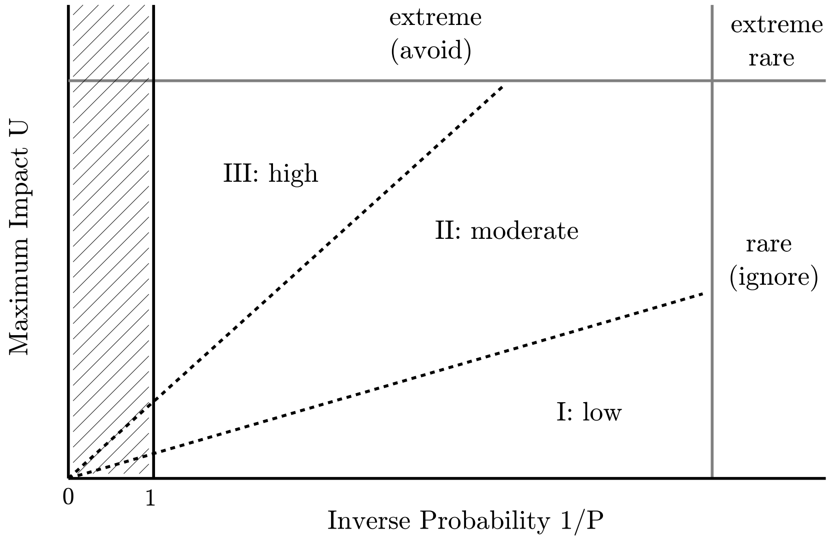

Figure 5.8: The Inverse Probability – Maximum impact graph divided into 6 areas. Areas I, II and III denote risks with increasing severity, while the areas labelled as "rare", "extreme" and "extreme rare" are self–explaining.

Suppose a risk at the point $Q = (1/P, U)$. Then $x = 1/P$ and $y = U$, therefore $y/x = P \cdot U = M$. Therefore, all events with equal expected value M lie on the same straight line $y = M \cdot x$. When moving on one such line to the right, one meets risks with greater maximum value, hence more unfavourable.

This graph makes it even easier to directly compare any two risks in terms of maximum impact (y–coordinate) as well as in terms of expected impact (slope of the straight line drown from the origin). Furthermore, the analyst can clearly distinguish among extreme, rare, and extreme rare events, by defining project–specific relevant bounds. For infrastructure projects based on scientifically and practically obtained design facts, only the areas I, II, III can be considered. The field of extreme rare event modelling is still at its infancy.

5.6 Risk grouping and dependence

Even in case the groups are not fully independent, it is be helpful to eliminate, at least, any functional relation among risks of distinct groups. The risks belonging to the same class have some common attributes; such attributes may be a common stimulus, underlying influence mechanism, source, expected time of occurrence, or common spatial properties.

As discussed in Chapter 4, there are three major issues regarding dependence; the analysts needs to assess, efficiently measure, and integrate dependence in the computational model. The first step in order to model dependence is the classification of risks into (mostly) independent groups. This can significantly reduce the amount of required calculations, e.g. reduce the dimension of the correlation matrix. Each group includes risks associated with tasks which, in turn, form parallel and mostly independent activity lines. In particular, risks identified within a certain workflow planned to take place in the same location and time phase are likely to possess a common cause, or even a direct influence on each other. For instance, adverse geological conditions will probably affect the advancement of two vicinal excavations, whereas a time delay within a sequence may affect more than one activity.

Risk dependencies can be identified by either diagnostic ("*Why did risk R_j occur?*") or predictive ("*What happens if risk R_j occurs?*") inference. The starting point should be at the risks that appear earlier in time. Usually, this questions generate a discussion which can lead, not only to the determination of dependencies, but also to the disclosure of new risks. For instance, a risk can "stimulate" the occurrence of another risk, otherwise considered dormant, or highly unlikely. When a risk is believed to have an effect, impact, or influence on another, then a possible dependency is identified. An investigation on common risk backround, mechanism, environment, or underlying process between two risks can reveal connections and associations. Dependencies can also be detected through common sense, or past experience.

The quantification process has also seen interesting developments. The following elicitation technique, due to vad Dorp and Duffey [vDD99], offers a methodology to quantify dependence: "*Suppose you were to know the exact value of the risk factor/cause R_j, what percentage of your original uncertainty in the assessment of the risk symptom R_k is explained?*". Then, 0% indicates independence and 100% perfect dependence.

Several techniques have also been proposed for subjectively assessing correlation values. The direct assessment method of simply asking for the correlation value has been found to be the best one [Rei00]. Next to that, various values have been proposed to express weak, moderate and strong correlation [BS99], [Ran00].

A detailed project plan, showing the risks in their proper location and time can provide a sound basis for grouping. Typically, risks are depicted as nodes and dependencies as edges or directed arcs. From the BBT risk assessment, a preliminary analysis resulted to 7 main groups, presented in Table (5.2).

The assessed dependencies for the groups "Arhental", "Aica", "Ampass", "Innsbruck" and "Mules" are depicted respectively in Figures (5.9), (5.10), (5.12), (5.13).

Risk group	Number of risks
Ampass	3
Innsbruck	2
Ahrental	7
Wolf	19
Mules	10
Aica	3
Fortezza	1
Overall risks	15
Ungrouped	29
sum	89

Table 5.2: The 7 identified risk groups in BBT.

The group "Fortezza" contains a single element, while the visualisation of "Wolf" is omitted, as having a quite complicated dependence structure.

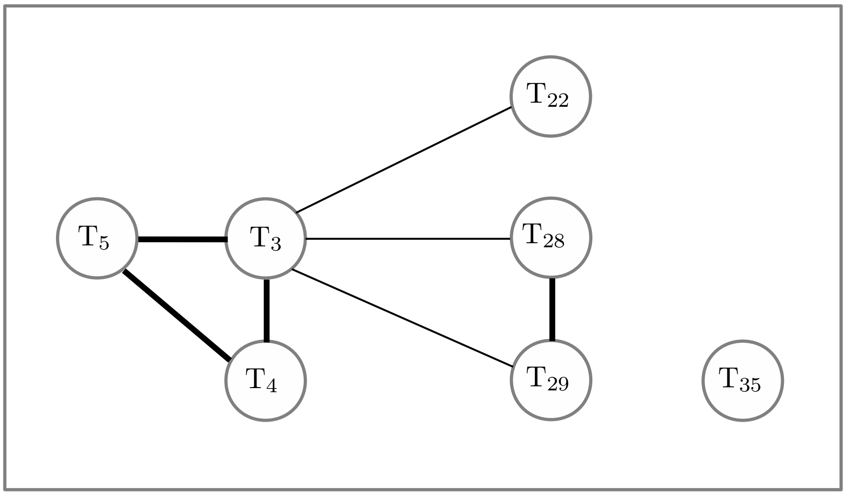

Figure 5.9: Dependencies within the Arhental group.

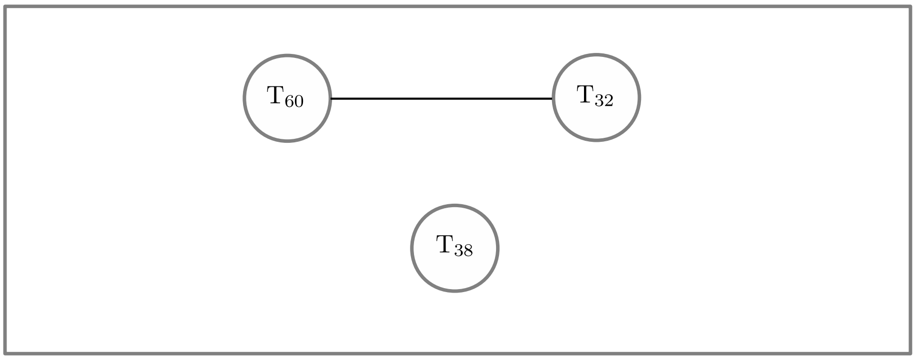

Figure 5.10: Dependencies within the Aica group.

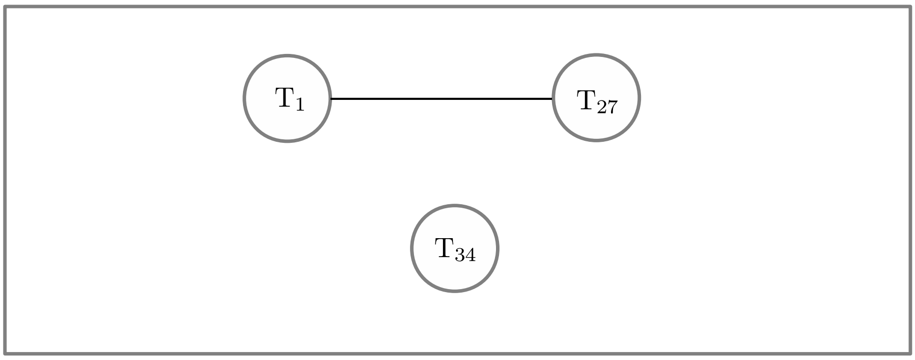

Figure 5.11: Dependencies within the Ampass group.

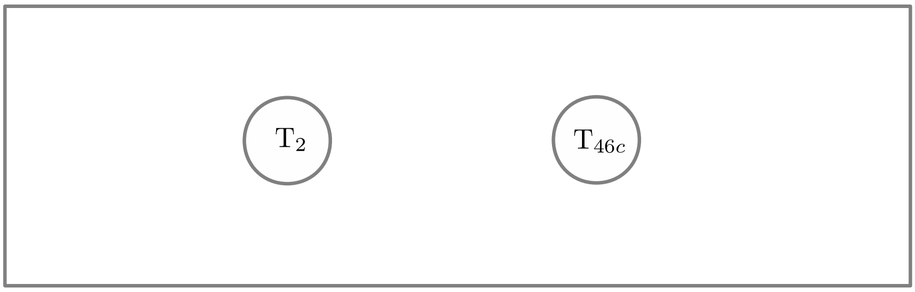

Figure 5.12: Dependencies within the Innsbruck group.

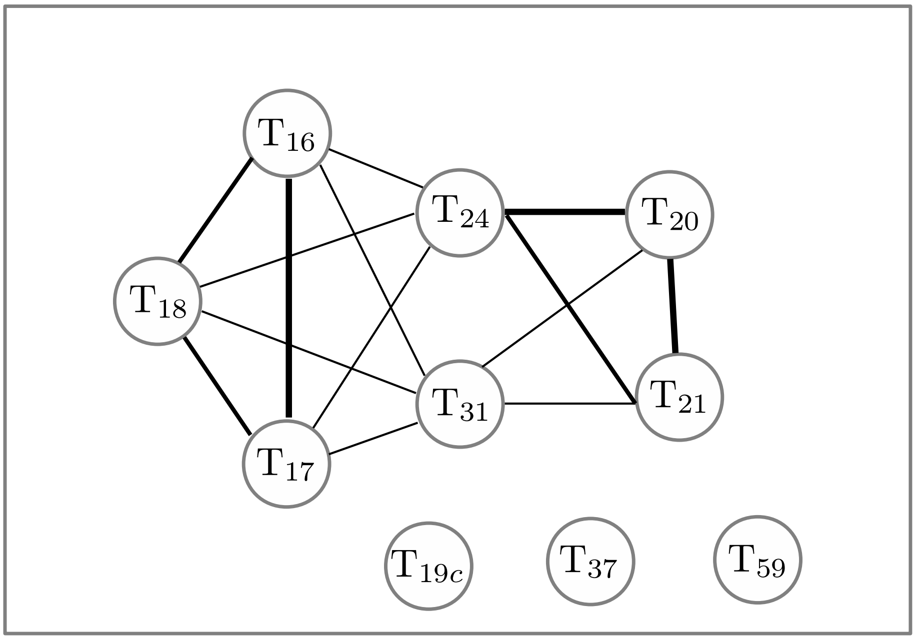

Figure 5.13: Dependencies within the Mules group.

Chapter 6

Application on the BBT project

6.1 Input data

The input marginal distributions are the beta densities, as presented in Section (5.4). The risk groups were given in Section (5.6), as well as the dependencies within groups (with one exception) in the form of graphs. In the present section the correlation matrices for each group are given; these matrices contain the rank correlations, as initially elicited. The interpretation of these values was described in Table (4.1). What follows directly is the tranformation of these matrices into usable correlation matrices in order to obtain model inputs, as described in Sections (4.7) and (4.8).

1. Ampass group

Risks: $\mathrm{T}_1, \mathrm{T}_{27}, \mathrm{T}_{34}$.

$$\Sigma_1 = \begin{array}{c c} & \begin{array}{cc} \mathrm{T}_1 & \mathrm{T}_{27} \end{array} \\ \begin{array}{c} \mathrm{T}_1 \\ \mathrm{T}_{27} \end{array} & \begin{pmatrix} 1 & 0.25 \\ & 1 \end{pmatrix} \end{array}$$

2. Innsbruck group

Risks: $\mathrm{T}_2, \mathrm{T}_{46c}$ (independent).

3. Ahrental group

Risks: $\mathrm{T}_3, \mathrm{T}_4, \mathrm{T}_5, \mathrm{T}_{22}, \mathrm{T}_{28}, \mathrm{T}_{29}, \mathrm{T}_{35}$.

$$
\Sigma_3 = \begin{array}{cc}
 & \begin{array}{cccccc} \mathrm{T}_3 & \mathrm{T}_4 & \mathrm{T}_5 & \mathrm{T}_{22} & \mathrm{T}_{28} & \mathrm{T}_{29} \end{array} \\
\begin{array}{c} \mathrm{T}_3 \\ \mathrm{T}_4 \\ \mathrm{T}_5 \\ \mathrm{T}_{22} \\ \mathrm{T}_{28} \\ \mathrm{T}_{29} \end{array} &
\begin{pmatrix}
1 & 0.75 & 0.75 & 0.25 & 0.25 & 0.25 \\
 & 1 & 0.75 & 0 & 0 & 0 \\
 & & 1 & 0 & 0 & 0 \\
 & & & 1 & 0 & 0 \\
 & & & & 1 & 0.75 \\
 & & & & & 1
\end{pmatrix}
\end{array}
$$

4. Wolf group

Risks: $\mathrm{T}_6, \mathrm{T}_7, \mathrm{T}_8, \mathrm{T}_9, \mathrm{T}_{10}, \mathrm{T}_{11}, \mathrm{T}_{12}, \mathrm{T}_{13}, \mathrm{T}_{14}, \mathrm{T}_{15}, \mathrm{T}_{23}, \mathrm{T}_{30}, \mathrm{T}_{36}, \mathrm{T}_{51}, \mathrm{T}_{52}, \mathrm{T}_{56}, \mathrm{T}_{57}, \mathrm{T}_{58}$.

$$
\Sigma_4 = \begin{array}{cc}
 & \begin{array}{cccccccccccccccccc} \mathrm{T}_6 & \mathrm{T}_7 & \mathrm{T}_8 & \mathrm{T}_9 & \mathrm{T}_{10} & \mathrm{T}_{11} & \mathrm{T}_{12} & \mathrm{T}_{13} & \mathrm{T}_{14} & \mathrm{T}_{15} & \mathrm{T}_{23} & \mathrm{T}_{30} & \mathrm{T}_{36} & \mathrm{T}_{51} & \mathrm{T}_{52} & \mathrm{T}_{56} & \mathrm{T}_{57} & \mathrm{T}_{58} \end{array} \\
\begin{array}{c} \mathrm{T}_6 \\ \mathrm{T}_7 \\ \mathrm{T}_8 \\ \mathrm{T}_9 \\ \mathrm{T}_{10} \\ \mathrm{T}_{11} \\ \mathrm{T}_{12} \\ \mathrm{T}_{13} \\ \mathrm{T}_{14} \\ \mathrm{T}_{15} \\ \mathrm{T}_{23} \\ \mathrm{T}_{30} \\ \mathrm{T}_{36} \\ \mathrm{T}_{51} \\ \mathrm{T}_{52} \\ \mathrm{T}_{56} \\ \mathrm{T}_{57} \\ \mathrm{T}_{58} \end{array} &
\begin{pmatrix}
1 & 0.75 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & 1 & 0.75 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & & & 1 & 0.75 & 0 & 0 & 0 & 0 & 0.5 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & & & & 1 & 0 & 0 & 0 & 0 & 0.5 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & & & & & 1 & 0.75 & 0 & 0 & 0.5 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & & & & & & 1 & 0 & 0 & 0.5 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & & & & & & & 1 & 0.75 & 0.5 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & & & & & & & & 1 & 0.5 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & & & & & & & & & 1 & 0 & 0 & 0.5 & 0.5 & 0.5 & 0.5 & 0 \\
 & & & & & & & & & & & 1 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0 \\
 & & & & & & & & & & & & 1 & 0 & 0 & 0 & 0 & 0 \\
 & & & & & & & & & & & & & 1 & 0.75 & 0 & 0 & 0 \\
 & & & & & & & & & & & & & & 1 & 0 & 0 & 0 \\
 & & & & & & & & & & & & & & & 1 & 0.75 & 0 \\
 & & & & & & & & & & & & & & & & 1 & 0 \\
 & & & & & & & & & & & & & & & & & 1
\end{pmatrix}
\end{array}
$$

5. Mules group

$\mathrm{T}_{16}, \mathrm{T}_{17}, \mathrm{T}_{18}, \mathrm{T}_{20}, \mathrm{T}_{21}, \mathrm{T}_{24}, \mathrm{T}_{31}, \mathrm{T}_{19c}, \mathrm{T}_{37}, \mathrm{T}_{53}, \mathrm{T}_{59}$

$$
\Sigma_5 = \begin{array}{cc}
 & \begin{array}{ccccccc} \mathrm{T}_{16} & \mathrm{T}_{17} & \mathrm{T}_{18} & \mathrm{T}_{20} & \mathrm{T}_{21} & \mathrm{T}_{24} & \mathrm{T}_{31} \end{array} \\
\begin{array}{c} \mathrm{T}_{16} \\ \mathrm{T}_{17} \\ \mathrm{T}_{18} \\ \mathrm{T}_{20} \\ \mathrm{T}_{21} \\ \mathrm{T}_{24} \\ \mathrm{T}_{31} \end{array} &
\begin{pmatrix}
1 & 0.75 & 0.50 & 0 & 0 & 0.25 & 0.25 \\
 & 1 & 0.50 & 0 & 0 & 0.25 & 0.25 \\
 & & 1 & 0 & 0 & 0.25 & 0.25 \\
 & & & 1 & 0.75 & 0.50 & 0.25 \\
 & & & & 1 & 0.50 & 0.25 \\
 & & & & & 1 & 0.25 \\
 & & & & & & 1
\end{pmatrix}
\end{array}
$$

6. Aica group

Risks: $\mathrm{T}_{32}, \mathrm{T}_{38}, \mathrm{T}_{60}$.

$$\Sigma_2 = \begin{array}{c c} & \begin{array}{cc} \mathrm{T}_{32} & \mathrm{T}_{60} \end{array} \\ \begin{array}{c} \mathrm{T}_{32} \\ \mathrm{T}_{60} \end{array} & \begin{pmatrix} 1 & 0.25 \\ & 1 \end{pmatrix} \end{array}$$

7. Fortezza group

Risks: T_{61}.

8. Overall and ungrouped

Risks: $\mathrm{T}_{2}, \mathrm{T}_{53}, \mathrm{T}_{59}, \mathrm{T}_{19c}, \mathrm{T}_{61}, \mathrm{T}_{25c}, \mathrm{T}_{26}, \mathrm{T}_{33c}, \mathrm{T}_{54}, \mathrm{T}_{34}, \mathrm{T}_{35}, \mathrm{T}_{37}, \mathrm{T}_{38}, \mathrm{T}_{39}, \mathrm{T}_{55}, \mathrm{T}_{40}, \mathrm{T}_{41}, \mathrm{T}_{42}, \mathrm{T}_{43}, \mathrm{T}_{44}, \mathrm{T}_{45c}, \mathrm{T}_{46c}, \mathrm{T}_{47}, \mathrm{T}_{48}, \mathrm{T}_{50}, \mathrm{V}_{1}, \mathrm{V}_{2}, \mathrm{V}_{3}, \mathrm{V}_{4}, \mathrm{V}_{6}, \mathrm{V}_{7}, \mathrm{V}_{8}, \mathrm{V}_{9}, \mathrm{V}_{10c}, \mathrm{V}_{11}, \mathrm{V}_{12}, \mathrm{V}_{13}, \mathrm{V}_{14}, \mathrm{V}_{15}, \mathrm{V}_{16c}, \mathrm{V}_{17c}, \mathrm{V}_{18}, \mathrm{V}_{19}, \mathrm{V}_{20}, \mathrm{V}_{21}, \mathrm{V}_{28}, \mathrm{V}_{29}, \mathrm{V}_{30}, \mathrm{V}_{31}, \mathrm{A}_{1a}, \mathrm{A}_{1b}, \mathrm{A}_{2}, \mathrm{A}_{3}, \mathrm{A}_{4}$

6.2 Computational setup

In the following, the steps for processing the given information is outlined.

Step 1

The data for individual cost elements, summarised at Table (5.1) are preprocessed. The last two columns containing the α and β parameters of the beta densities, are calculated according to Formulas (3.17) and (3.18).

Name	**minimum**	**mode**	**maximum**	**confidence**	**corrected confidence**	α	β
T–1	0	1.5	3.0	0.42	0.42	2.3	2.3
T–2	0	1.0	5.0	0.42	0.50	2.8	8.0
T–3	0	1.2	2.0	0.42	0.43	1.9	1.6
T–4	0	3.0	5.0	0.42	0.43	1.9	1.6

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Table 6.1 – *...continued from previous page*

Name	minimum	mode	maximum	confidence	corrected confidence	α	β
T–5	0	4.0	20.0	0.42	0.50	2.8	8.0
T–6	0	0.4	2.0	0.42	0.50	2.8	8.0
T–7	0	0.8	4.0	0.42	0.50	2.8	8.0
T–56	0	5.0	10.0	0.42	0.42	2.3	2.3
T–57	0	10.0	20.0	0.42	0.42	2.3	2.3
T–8	0	28.0	40.0	0.42	0.46	1.6	1.2
T–9	0	35.0	50.0	0.42	0.46	1.6	1.2
T–10	0	24.0	30.0	0.42	0.50	1.3	1.1
T–11	0	40.0	50.0	0.42	0.50	1.3	1.1
T–12	0	0.8	1.0	0.42	0.50	1.3	1.1
T–13	0	1.6	2.0	0.42	0.50	1.3	1.1
T–14	0	0.4	1.0	0.42	0.43	2.6	3.5
T–53	0	0.8	2.0	0.42	0.43	2.6	3.5
T–15	0	20.0	50.0	0.42	0.43	2.6	3.5
T–51	0	2.9	7.3	0.42	0.43	2.6	3.5
T–52	0	4.3	10.7	0.42	0.43	2.6	3.4
T–58	0	16.0	20.0	0.42	0.50	1.3	1.1
T–59	0	32.0	40.0	0.42	0.50	1.3	1.1
T–16	0	21.0	30.0	0.42	0.46	1.6	1.2
T–17	0	14.0	20.0	0.42	0.46	1.6	1.2
T–18	0	9.0	30.0	0.42	0.46	2.7	5.0
T–19c	-40.0	-8.0	0	0.42	0.50	1.3	1.1
T–20	0	24.0	30.0	0.42	0.50	1.3	1.1
T–21	0	32.0	40.0	0.42	0.50	1.3	1.1
T–60	0	1.5	5.0	0.42	0.46	2.7	5.0
T–61	0	8.0	20.0	0.42	0.43	2.6	3.5
T–22	0	3.0	10.0	0.42	0.46	2.7	5.0
T–23	0	6.0	20.0	0.42	0.46	2.7	5.0
T–24	0	6.0	20.0	0.42	0.46	2.7	5.0
T–25c	-49.0	-4.9	0	0.42	0.54	1.1	1.0^{+}
T–26	0	24.0	80.0	0.42	0.46	2.7	5.0
T–27	0	0.3	1.0	0.42	0.46	2.7	5.0
T–28	0	2.5	5.0	0.42	0.42	2.3	2.3

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Table 6.1 – *...continued from previous page*

Name	**minimum**	**mode**	**maximum**	**confidence**	**corrected confidence**	α	β
T–29	0	3.0	10.0	0.42	0.46	2.7	5.0
T–30	0	4.0	10.0	0.42	0.43	2.6	3.5
T–31	0	0.6	2.0	0.42	0.46	2.7	5.0
T–32	0	3.0	10.0	0.42	0.46	2.7	5.0
T–33c	-6.1	-1.2	0	0.42	0.50	1.3	1.1
T–54	0	3.7	12.3	0.42	0.46	2.7	5.0
T–34	0	0.6	2.0	0.42	0.46	2.7	5.0
T–35	0	1.5	5.0	0.42	0.46	2.7	5.0
T–36	0	3.0	10.0	0.42	0.46	2.7	5.0
T–37	0	1.5	5.0	0.42	0.46	2.7	5.0
T–38	0	3.0	10.0	0.42	0.46	2.7	5.0
T–39	0	8.0	20.0	0.42	0.43	2.6	3.5
T–55	0	16.0	40.0	0.42	0.43	2.6	3.5
T–40	0	9.0	30.0	0.42	0.46	2.7	5.0
T–41	0	4.9	49.0	0.42	0.54	2.8	17.5
T–42	0	1.8	18.4	0.42	0.54	2.8	17.9
T–43	0	4.2	42.4	0.42	0.54	2.8	17.6
T–44	0	7.2	18.0	0.42	0.43	2.6	3.5
T–45c	-49.0	-4.9	0	0.42	0.54	1.1	1.0^{+}
T–46c	-15.2	-7.6	0	0.42	0.42	2.3	2.3
T–47	0	15.5	51.8	0.42	0.46	2.7	5.1
T–48	0	3.3	11.0	0.42	0.46	2.7	5.0
T–50	0	9.6	24.0	0.42	0.43	2.6	3.5
V–1	0	0.0	0.0	NA	NA	NA	NA
V–2	0	28.1	56.3	0.42	0.42	2.3	2.3
V–3	0	0.6	5.7	0.42	0.54	2.8	16.5
V–4	0	0.6	1.2	0.42	0.42	2.3	2.3
V–6	0	7.5	37.7	0.42	0.50	2.8	8.1
V–7	0	8.0	16.0	0.42	0.42	2.3	2.3
V–8	0	3.2	10.5	0.42	0.46	2.7	5.0
V–9	0	27.0	54.0	0.42	0.42	2.3	2.3
V–10c	-37.7	-7.5	0	0.42	0.50	1.3	1.1
V–11	0	6.4	16.0	0.42	0.43	2.6	3.5

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Table 6.1 – ...*continued from previous page*

Name	**minimum**	**mode**	**maximum**	**confidence**	**corrected confidence**	α	β
V–12	0	15.1	37.7	0.42	0.43	2.6	3.4
V–13	0	3.2	8.0	0.42	0.43	2.6	3.5
V–14	0	3.8	37.7	0.42	0.54	2.8	17.3
V–15	0	8.0	40.0	0.42	0.50	2.8	8.0
V–16c	-37.7	-4.0	0	0.42	0.54	1.1	1.0^{+}
V–17c	-8.0	-2.0	0	0.42	0.48	1.4	1.1
V–18	0	0.0	0.0	NA	NA	NA	NA
V–19	0	0.3	1.7	0.42	0.51	2.8	9.3
V–20	0	1.0	5.1	0.42	0.50	2.8	8.2
V–21	0	0.3	1.7	0.42	0.51	2.8	9.3
V–28	0	15.1	37.7	0.42	0.43	2.6	3.4
V–29	0	4.4	11.0	0.42	0.43	2.6	3.5
V–30	0	5.4	27.0	0.42	0.50	2.8	8.0
V–31	0	6.4	12.8	0.42	0.42	2.3	2.3
A–1a	0	1.7	8.5	0.42	0.50	2.8	8.0
A–1b	0	6.6	33.0	0.42	0.50	2.8	8.0
A–2	0	10.0	20.0	0.42	0.42	2.3	2.3
A–3	0	20.0	100.0	0.42	0.50	2.8	8.0
A–4	0	8.0	20.0	0.42	0.43	2.6	3.5

Table 6.1: List of the risks and opportunities with assessed minimum, mode, maximum and confidence values. The last two columns include the calculated α and β parameters of the beta densities. The parameters denoted as 1.0^{+} are greater than 1 but rounded to the first digit in the table.

Step 2

The mean (average) values for the individual cost elements, as well as the probabilities P and P^* (as described in Section (5.5)) are calculated. The generated risk matrices are presented at the end of the section in Figures (6.15) to (6.20). The matrices are given for both the cases where E or M is assumed.

Step 3

The assessed correlation matrices given in Section (6.1) are trasnformed to feasible ones, according to the process described in Section (4.7) and demonstrated in EXAMPLE (4.2). Firstly, the matrix entries are tranformed from rank into linear correlations, according to Formula (4.18). The derived matrices are tranformed using the spectral decomposition technique into feasible (positive–definite) correlation matrices, to be used as simulation inputs.

Step 4

The Cholesky decomposition of input matrices is performed. A large number of variates is simulated from the correlated beta densities, according to the gaussian copula procedure described in Section (4.8) and demonstrated in EXAMPLE (4.3).

Step 5

The results are processed to yield summary statistics for the total cost.

6.3 Total cost estimation

The procedure described in the preceding section yields the following results:

- Sum of modal values: 638.0
- Sum of maximum values: 1666.2

  It is clear that the maximum value is extremely conservative as a design value.

Independence case

For comparison, the total cost probability density is also calculated for the case where all risks are assumed independent. Under this assumption, the values are:

- Mean (average) value: 540.0
- Standard deviation: 54.2
- 95% Quantile: 629.3
- Expected Shortfall: 651.0

Dependence case

The total cost density is calculated for the general case where correlations are considered:

- Mean (average) value: 540.0
- Standard deviation: 65.5
- 95% Quantile: 646.8
- Expected Shortfall: 673.6

The latter result (673.6) is the final proposed figure. This value is not significantly larger than the mean (540.0) or the sum of modal values (638.0). This can be explained mainly by three reasons:

1. The minimum values were all set to zero, since no assessment was at hand. Unrealistically low impacts tend to pull the estimates to lower levels.
2. Many of the input distributions are highly skewed, due to large epistemic uncertainty. Hence, a large area near the tail is assigned to low probabilities.
3. A large number of dependencies remains unassessed in the current preliminary phase.

The graphs (6.1), (6.2) show the PDF's for the two simulated cases (independence and dependence).

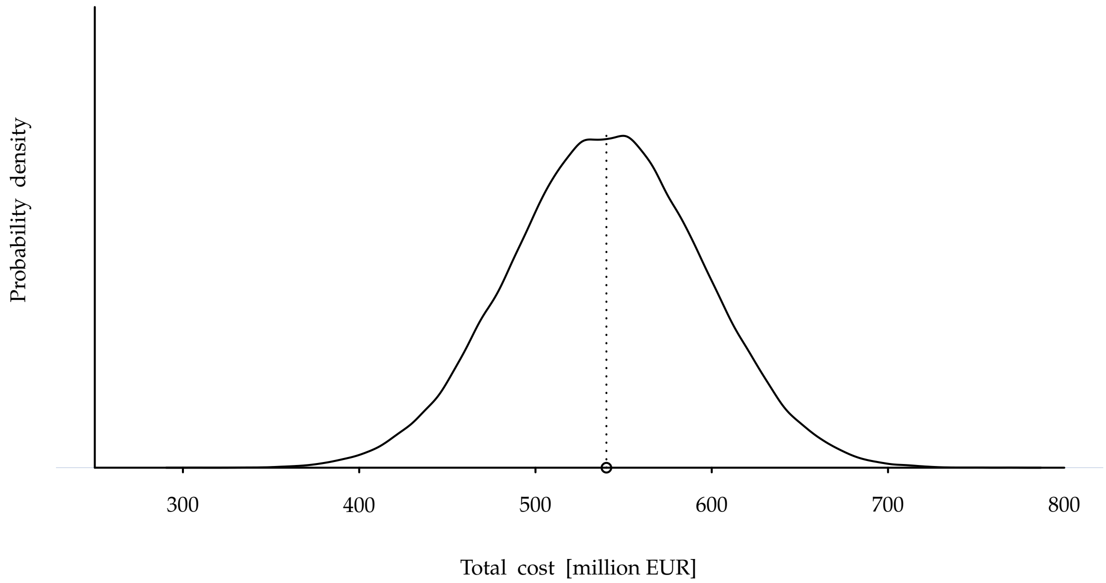

Figure 6.1: Probability density of the total risk cost for the independence case.

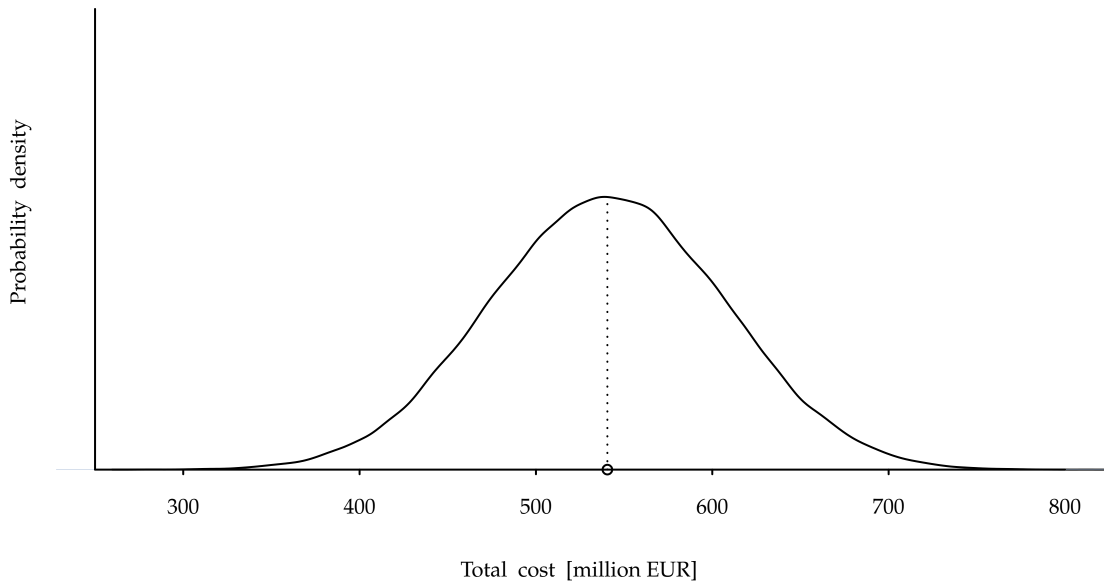

Figure 6.2: Probability density of the total risk cost for the dependence case.

6.4 Sensitivity analysis

The influence of epistemic uncertainty, which is described by the coefficient c (lack of confidence on the assessment) is shown in the following graphs. Figures (6.3) and (6.4) depict the probability density for the three levels of c (0.25, 0.33, 0.42) in the independence and the dependence case, respectively. Figures (6.5) to (6.12) show how the mean value, the 95% quantile and the expected shortfall of the estimated total cost density are influenced as the parameter c covers the feasible range. In particular, Figures (6.7), (6.8), (6.11) and (6.12) show that the dispersion of the total cost density incereases in a linear manner with the parameter c, justifying the selection of this factor.

Figures (6.5), (6.6), (6.9) and (6.10) show that when the lack of confidence is not corrected, the 95% and the expected shortfall remain constant within the interval $[0.25, 0.42]$ while the mean value is dropping. This behaviour is not realistic. On the contrary, when the lack of confidence is corrected as explained in Section (3.7), the 95% and the expected shortfall increase within the interval $[0.25, 0.42]$ while the mean value remains constant, as desired. Moreover, Figures (6.6) and (6.10) show that the selection of the interval $[0.25, 0.42]$ is justified, since the three aformentioned metrics have a consistent behaviour within this range.

Figures (6.13) and (6.14) depict the probability density of the total risk cost for the independence case, for both corrected and uncorrected lack of confidence, when the latter value is fixed at $c = 0.42$. The corrected case appears to be slightly less conservative, however when the expected shortfall is calculated, the results show unnoticeable deviation.

A sensitivity analysis on the uncertainty of the selected dependence values is not carried out since, as it appears in the results given in the previous section, the inclusion of dependence does not show a large influence at the current state of the analysis. However, if the assessment of dependence were complete, this investigation should be part of the sensitivity analysis.

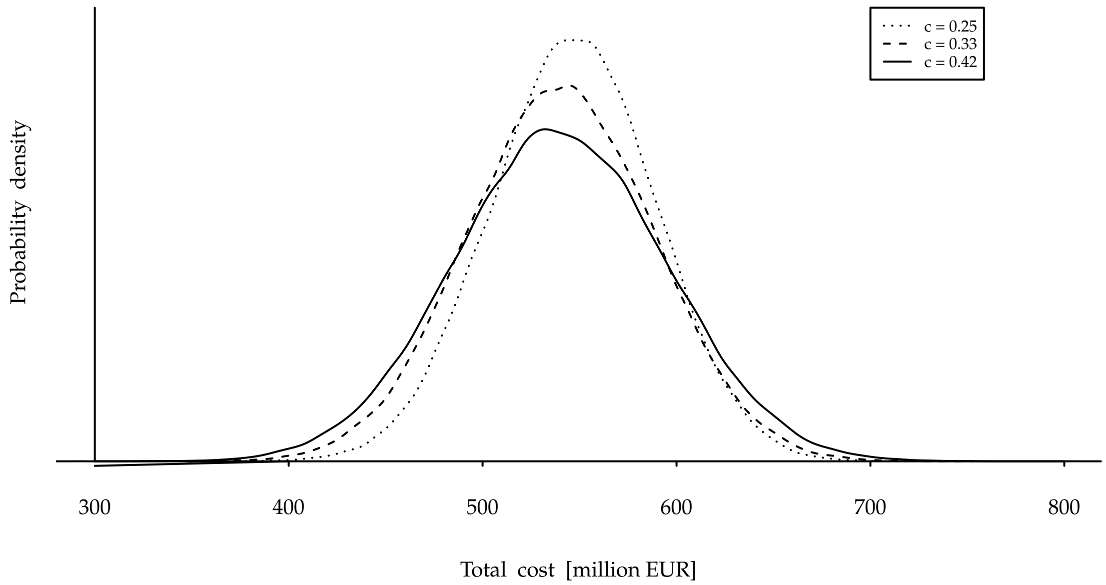

Figure 6.3: Probability density of the total risk cost for the independence case, and the three levels for the lack of confidence.

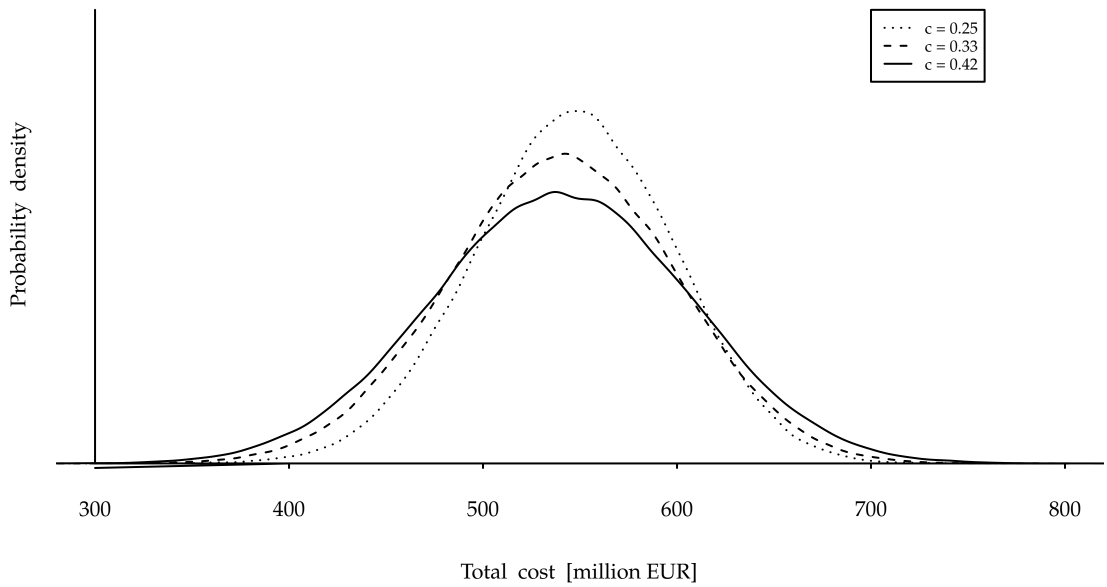

Figure 6.4: Probability density of the total risk cost for the dependence case, and the three levels for the lack of confidence.

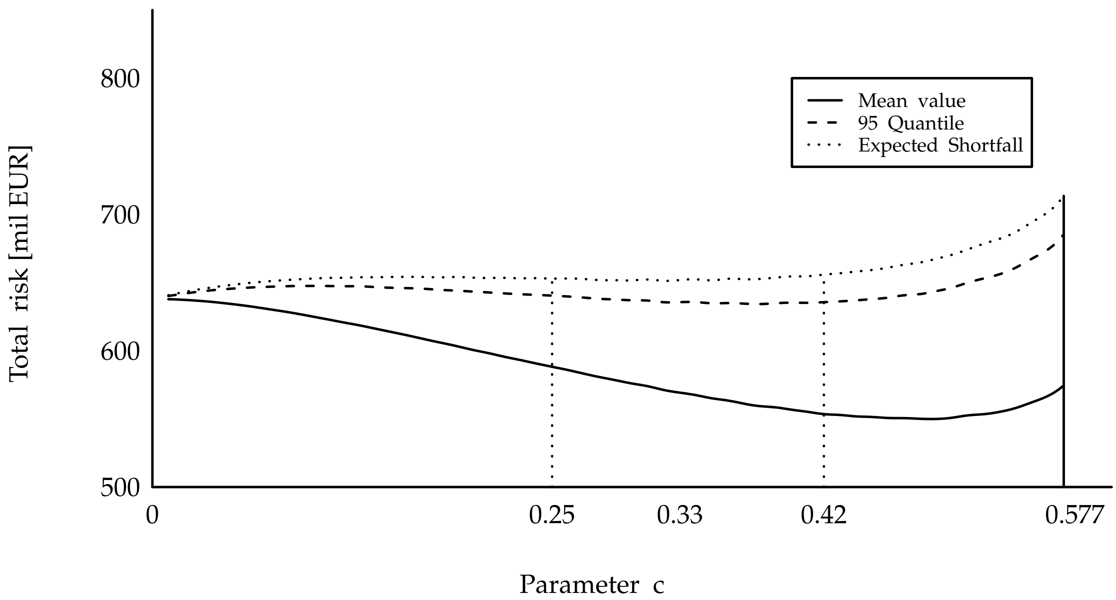

Figure 6.5: Influence of the selection of parameter c on the three metrics: mean value, 95% quantile and expected shortfall (independence case). Confidence was not corrected.

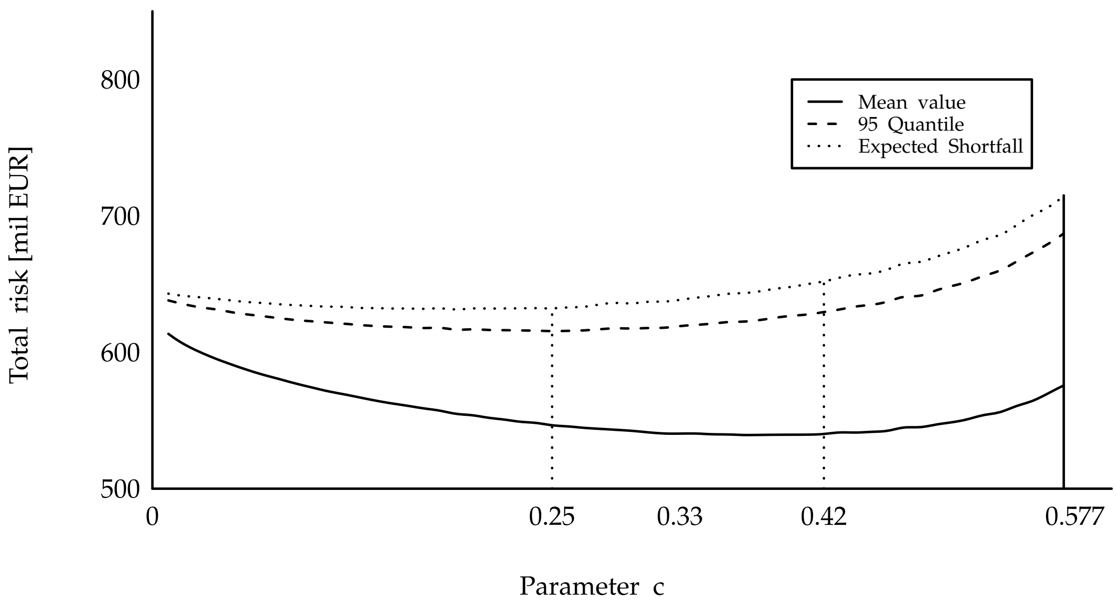

Figure 6.6: Influence of the selection of parameter c on the three metrics: mean value, 95% quantile and expected shortfall (independence case). Confidence was corrected as explained in Section (3.7).

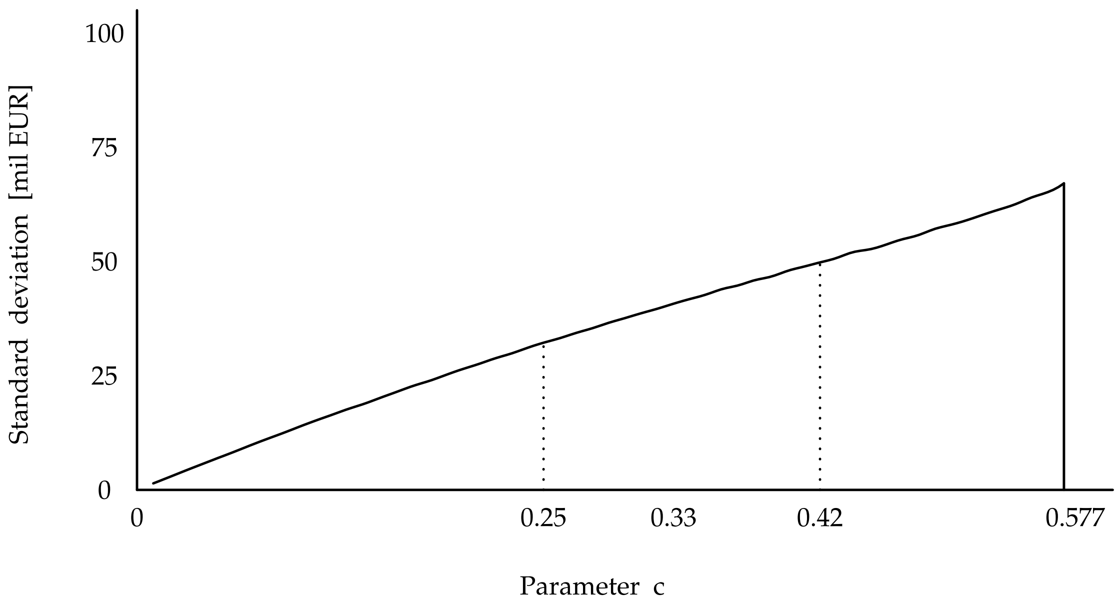

Figure 6.7: Influence of the selection of parameter c on the standard deviation (independence case). Confidence was not corrected.

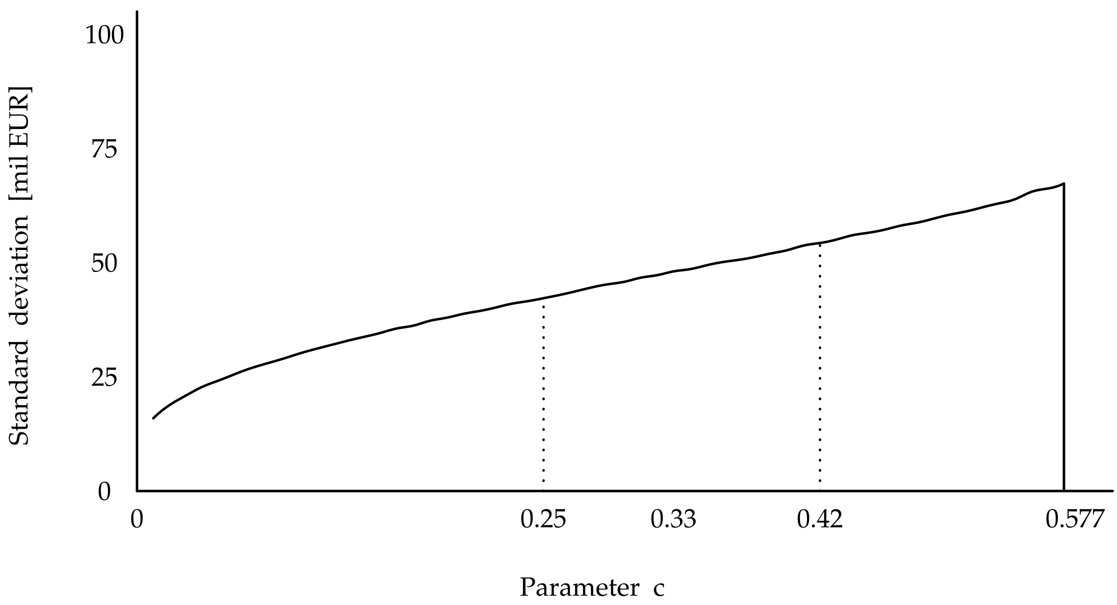

Figure 6.8: Influence of the selection of parameter c on the standard deviation (independence case). Confidence was corrected as explained in Section (3.7).

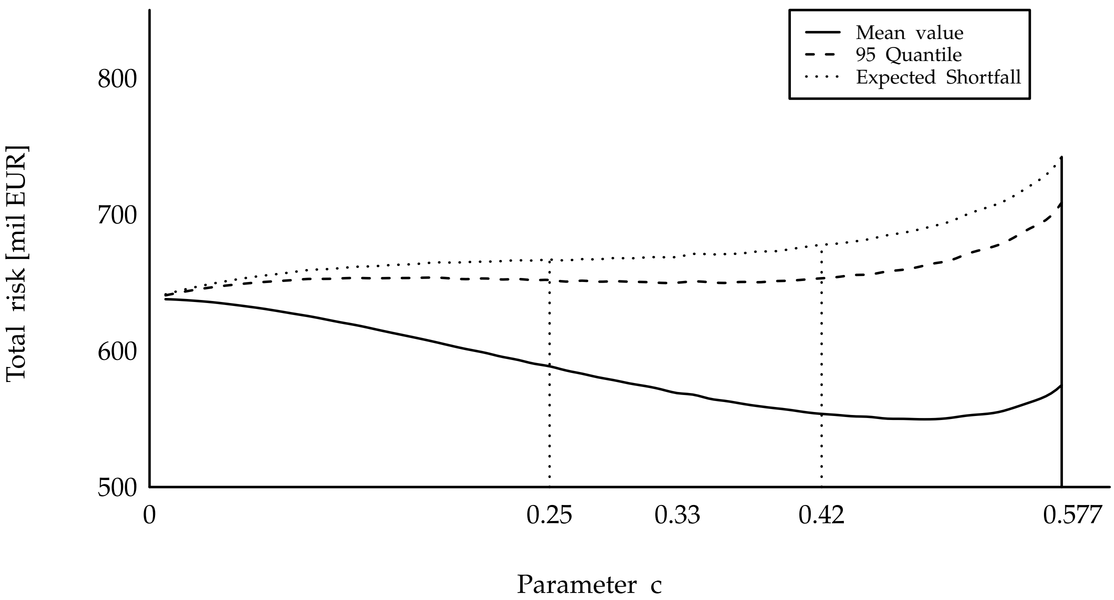

Figure 6.9: Influence of the selection of parameter c on the three metrics: mean value, 95% quantile and expected shortfall (dependence case). Confidence was not corrected.

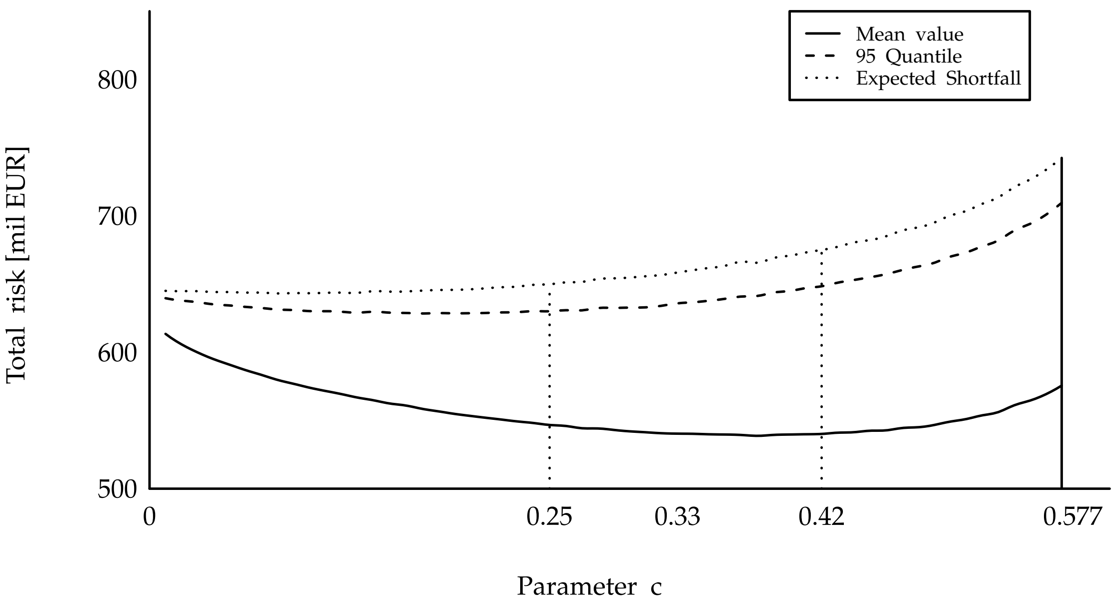

Figure 6.10: Influence of the selection of parameter c on the three metrics: mean value, 95% quantile and expected shortfall (dependence case). Confidence was corrected as explained in Section (3.7).

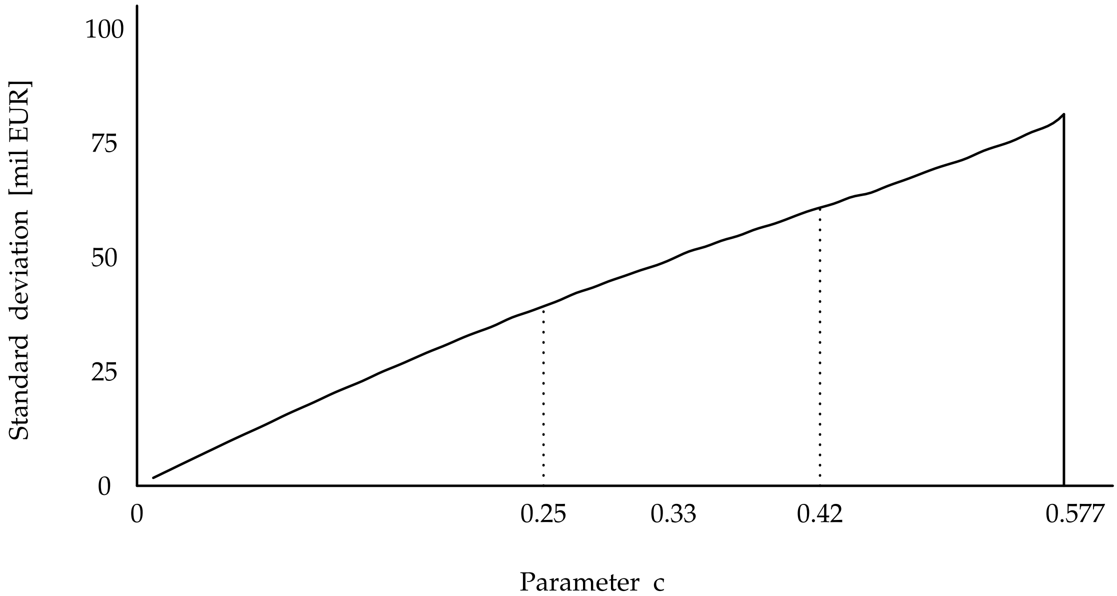

Figure 6.11: Influence of the selection of parameter c on the standard deviation (dependence case). Confidence was not corrected.

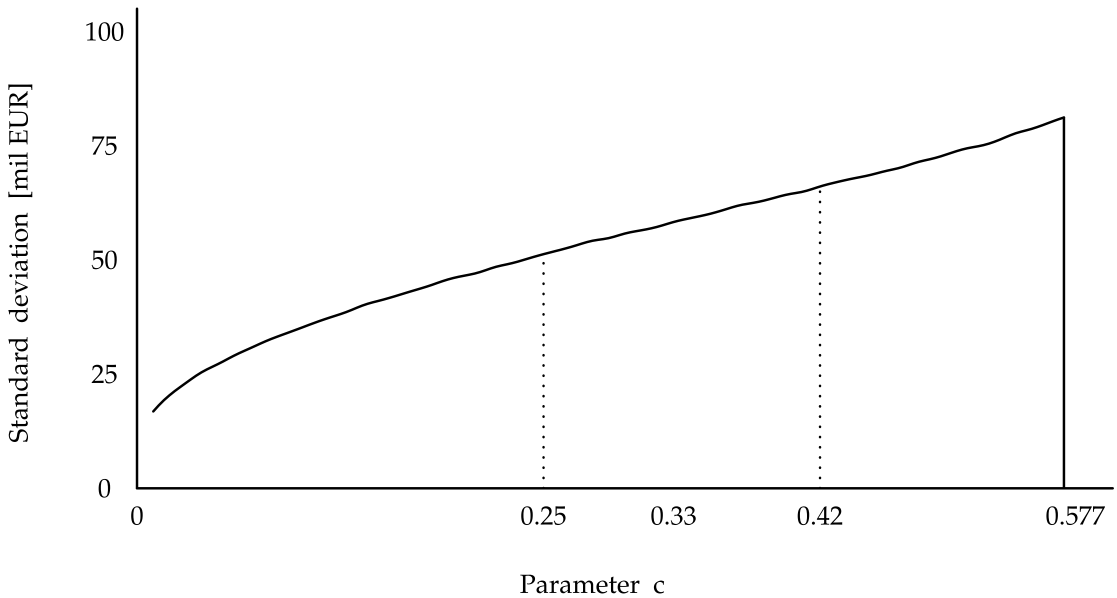

Figure 6.12: Influence of the selection of parameter c on the standard deviation (dependence case). Confidence was corrected as explained in Section (3.7).

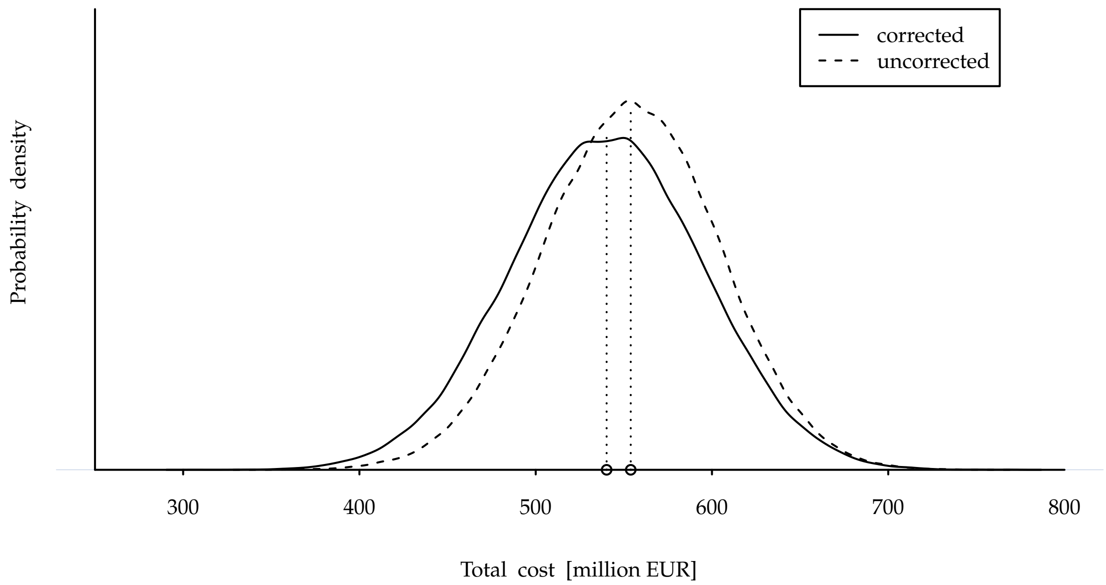

Figure 6.13: Probability density of the total risk cost for the independence case, for both corrected and uncorrected lack of confidence (fixed at $c = 0.42$).

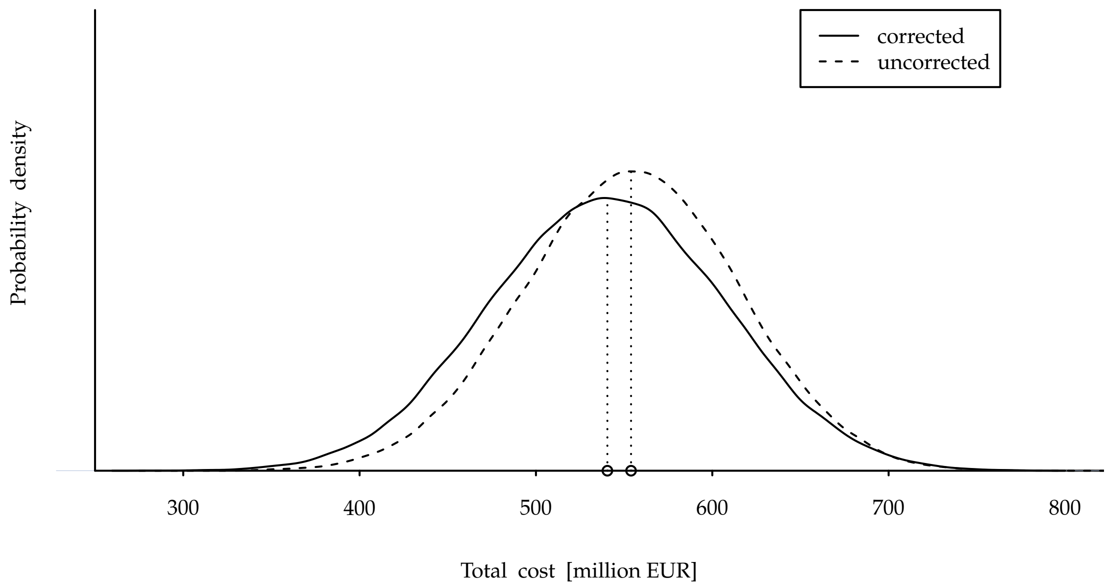

Figure 6.14: Probability density of the total risk cost for the dependence case, for both corrected and uncorrected lack of confidence (fixed at $c = 0.42$).

6.5 Risk matrices

Figures (6.15) to (6.20) depict the assessed risks, using the three schemes described in Section (5.5). The three approaches provide the same visual information, but in different ways.

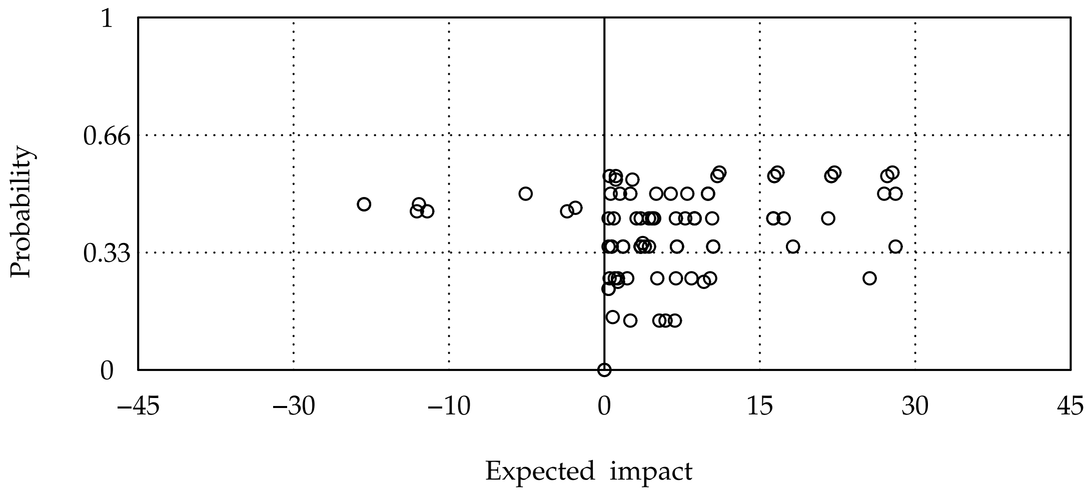

Figure 6.15: Expected Impact – Probability graph.

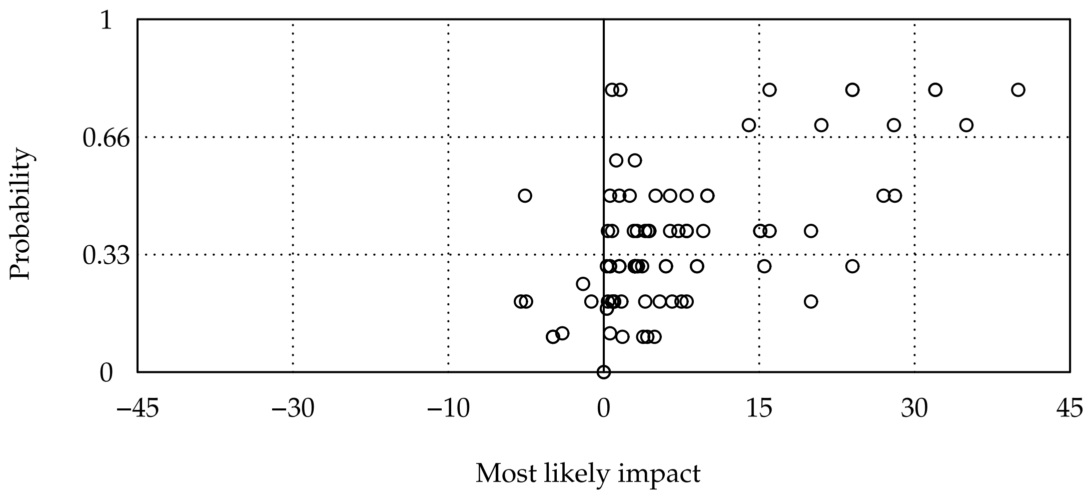

Figure 6.16: Most likely Impact – Probability graph.

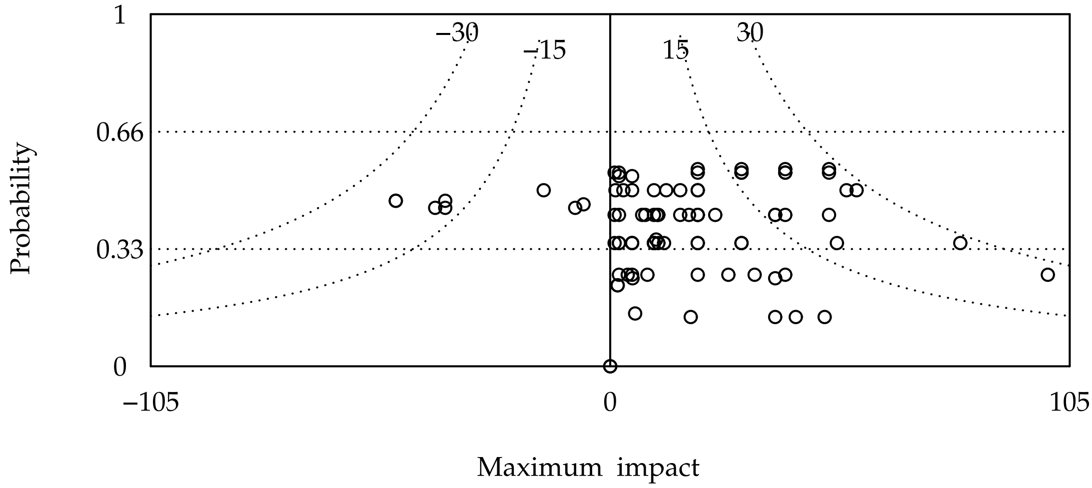

Figure 6.17: Maximum Impact – Probability graph. The probabilities are calculated as U/E.

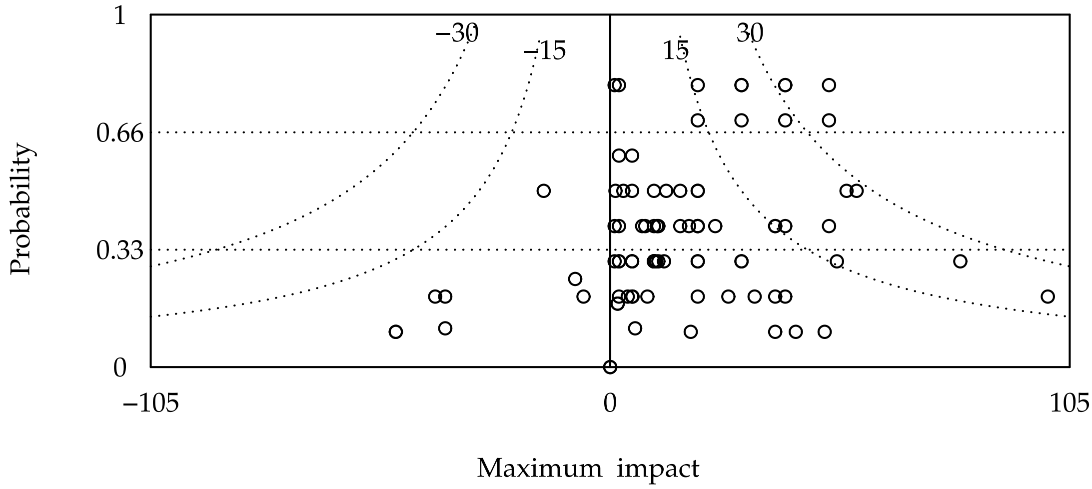

Figure 6.18: Maximum Impact – Probability graph. The probabilities are calculated as U/M.

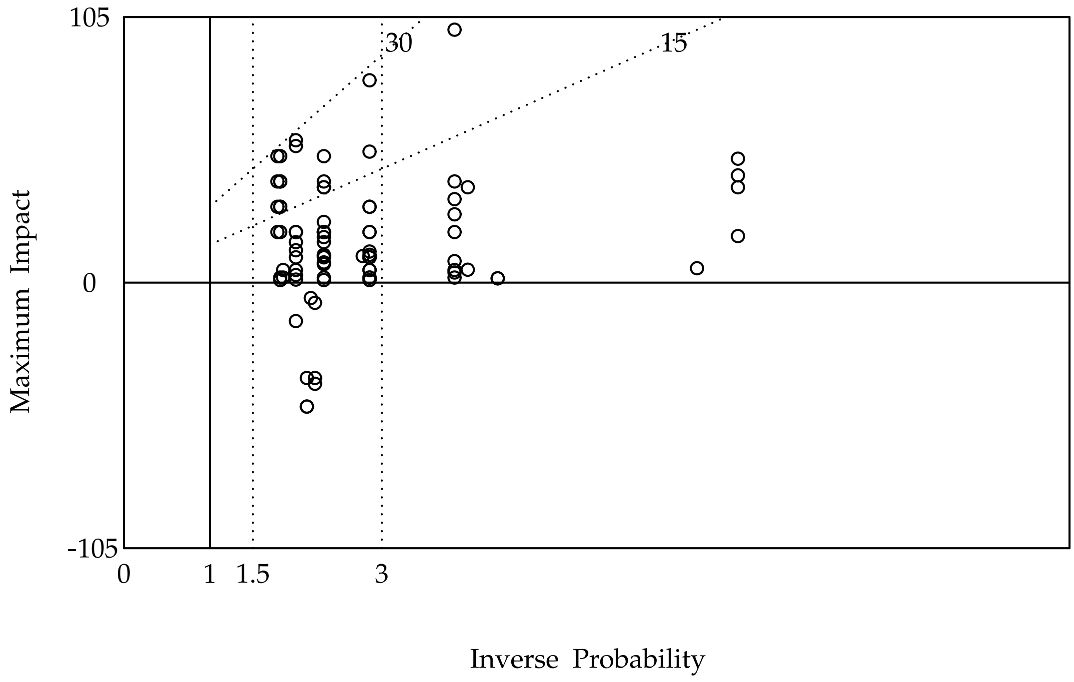

Figure 6.19: Inverse Probability – Maximum Impact graph. The probabilities are calculated as U/E.

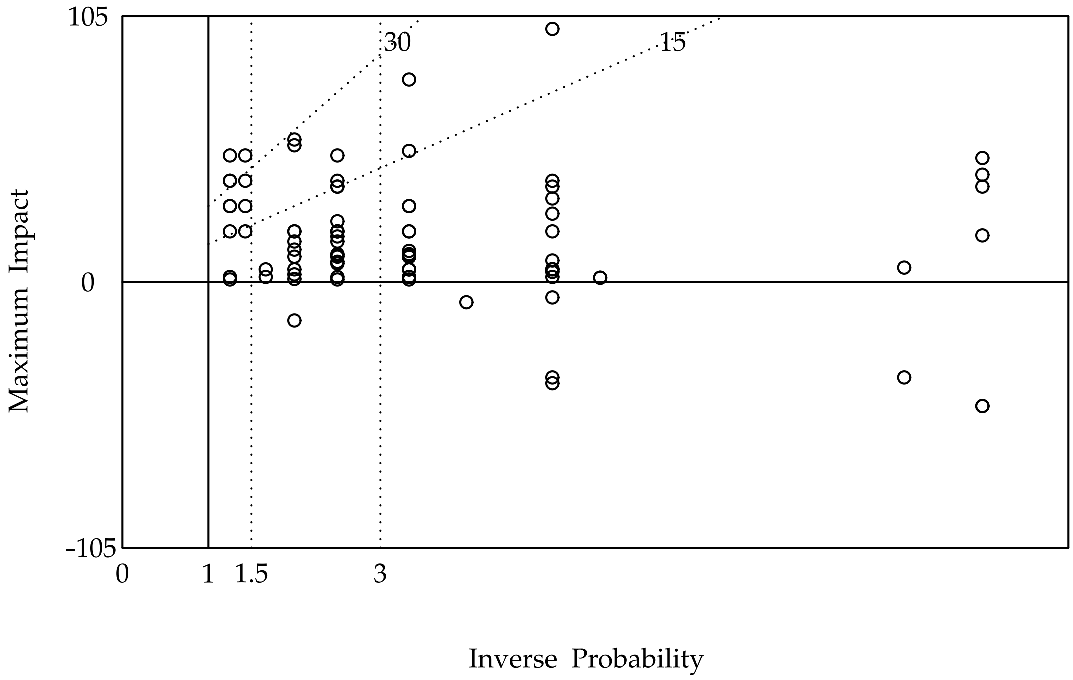

Figure 6.20: Inverse Probability – Maximum Impact graph. The probabilities are calculated as U/M.

Chapter 7

Synopsis

7.1 Overview

Complex, unique and demanding tunnelling projects depend on largely unknown geological conditions [PS06], and on political and managerial decisions made within a volatile economic environment. Moreover, the measures used to evaluate the performance and success of such projects relate to several different factors, such as quality, safety, functionality, duration, and cost [OO10]. This enormous complexity of mega–projects often results to interdependent, overlapping, or even conflicting tasks and processes. For instance, project schedules dictate resource requirements, while constraints on resources dictate project schedules [NS08, p. 289]. Mutual influence exists likewise between schedules and risks. Therefore, there is a need to formalise risk assessment processes, which naturally leads to risk quantification techniques.

Several methods have been developed for performing quantitative risk analysis in construction [CYC09]. In the present work, the probabilistic cost analysis (PCA) approach was taken for the estimation of total construction cost owing to risks. This methodology can offer significant advantages over traditional empirical approaches, which often yield severe cost underestimations in large infrastructure projects [FHB02]. Producing estimates by scaling an analogy project [NS08, p. 300] does not usually suffice, due to the uniqueness of the undertaking and the singularity of conditions. Still, there is no consensus on whether this excesses can be attributed to poor engineering and flawed risk management, or to unforeseen events of large scale.

Cost underestimations cannot be justified only upon the natural inability to foresee and capture every possible cost element. Once the limitations in the predictive process be acknowledged, realistic yet safer economic values, larger than the expected one, can be estimated. The limitations can be indeed translated into and treated as uncertainties within consistent computational models. This is the main purpose of the present research.

The proposed method comprises three steps. The first step, discussed in Chapter 3, is the representation of identified risks as individual cost elements, by utilising quantitative and qualitative information from expert judgements. Since the marginal distributions alone can give no information about the joint behaviour, dependencies among risks need also be considered. This is the subject of Chapter 4, where dependence is handled by means of Kendall's concordance coefficients. The significant effect of dependence on cost estimates, postulated by many sereaschers, is indeed shown in Chapter 6. The third step in the proposed approach is the integration of cost elements and correlations into a computational framework.

7.2 Uncertainties in the studied process

There are different types of uncertainties interfering in the described probabilistic setup. Uncertainties need to not only be measured but also be propagated through the process. Firstly, there are variable risks for which a quantitative assessment in terms of a single figure is largely arbitrary, but variability can be estimated (known unknowns). Moreover, there may be risks that are not identified (unknown unknowns). For the identified risks, the assessment of parameters (minimum, most likely, maximum value) is based on expert opinions, inavoidably hindered by ignorance and distorted by bias.

The reliability of information sources is related to uncertainty; an input value may ignore existing, or induce non–existing information. When elicited, subjective (judgemental, personal, or knowledge–based) probabilities are prone to inconsistencies, due to arbitrariness in judgement quantifications. Although probabilities can be intuitively attributed to even highly uncertain events [OO05], this alone does not provide to these assessed data any real predictive value. Finally, the veracity of an assessed probability of one–off events cannot be determined from subsequent observations [OBD+06].

In general, increased knowledge does not directly or necessarily lead to assessment improvements [AP98]. Data in historical records can become obsolete in the light of newer technologies and materials [Yan05]. Likewise, experience from similar undertakings can prove insufficient in terms of predictive potetial. For instance, technical problems in tunnelling trivially addressed at shallow depth, can prove disastrous at significal depth depending on hydrogeological conditions [Wag12].

Uncertainty models can yield huge deviations from reality. Limitations in the analyst's knowledge and deliberate introduced simplifications can affect the calculated quantities [NA03]. Oversimplifications in PCA (use of triangular distributions, omission of dependencies, no providence for confidence levels) are frequent [KAE04]. On the downside, adherence to strict mathematical methods (such as the maximum entropy principle) for translating assessments into model inputs, can turn out to be problematic [dRDT08], especially under large epistemic uncertainty. Next to that, the assessor may "over–determine" the model by assessing more aspects than are mathematically required [Dic80].

Multivariate risk assessment often suffers by a partial or complete lack of information on the nature and the magnitude of the interaction between several variables [GGGR09]. There is a general tendency to overlook correlations, due to the difficulty to detect, measure and integrate this type of information. Moreover, dependencies are not relevant when single point estimates are used [BS99], which is frequently the case.

7.3 Conclusions

A probabilistic cost analysis setup is a complex simulation process consisting of the following steps [CYC09]: (1) identification of random variables, (2) quantification of dependencies, (3) generation of random variates, (4) calculation of the desired output, (5) calculation of useful statistics and generation of graphs. The credibility of a computational model, demonstrated through verification and validation, measures the extent to which simulation results can be analysed with confidence to represent the phenomenon of interest with a degree of accuracy consistent with the intended use of the model [HH04]. Hence, in cases such as a cost estimation of a complex one–off project, prediction credibility is a highly challenging task.

Ignorance is rather an attitude than a fact; there is much information confined in expertise and experience. The aim of an analysis is to utilise all the relevant information in an unbiased way [OBD$^+$06]. In the present analysis, an integrated method is demostrated that uses existent information in order to produce economic estimates. Moreover, it is attempted to explore the nature and the influence of aleatory and epistemic uncertainties on the final output, so as to evaluate the credibility of those estimates. Also, the developed model is flexible, and can be adapted in the light of new information with regard to the individual cost elements, as well as to the dependences among these elements.

Appendix A

Statistical distributions

All random variables mentioned in the present study are defined on a common probability space $(\Omega, \mathcal{A}, \mathbb{P})$ and assume real values. The present Appendix follows the notation, as given in [DKLM07].

Let $\mathbb{R}$ denote the ordinary real line $(-\infty, +\infty)$ and $\overline{\mathbb{R}}$ the extended real line $[-\infty, +\infty]$. A distribution function is a nondecreasing function F with domain $\overline{\mathbb{R}}$ such that $F(-\infty) = 0$ and $F(+\infty) = 1$. Once a random variable X has been defined, the (cumulative) distribution function, abreviated as CDF, of X, is the function $F_X : \mathbb{R} \to [0, 1]$ defined by:

$$F_X(x) = P(X \leq x)$$

Any function defined as above can be shown to be a distribution function. A random variable X is continuous if for some function $f_X : \mathbb{R} \to [0, 1]$ and for any numbers a and b with $a \leq b$,

$$P(a \leq x \leq b) = \int_a^b f_X(x)\mathrm{d}x$$

The function f_X has to satisfy $f_X(x) \geq 0$ for all x and $\int_{-\infty}^{+\infty} f_X(x)\mathrm{d}x = 1$. Then f_X is called the probability density function, abbreviated as PDF, of X. The CDF and the PDF are related according to the following formulas:

$$F_X(x) = \int_{-\infty}^{x} f_X(t)\mathrm{d}t$$

$$f_X(x) = \frac{\mathrm{d}}{\mathrm{d}x} F_X(x).$$

The *expectation* (*expected value*, *mean* or *first moment* of X) of a continuous random variable X with probability density function f is the quantity:

$$\mathrm{E}[X] = \int_{-\infty}^{\infty} x f(x) \mathrm{d}x$$

The *variance* of a continuous random variable X with expectation $\mathrm{E}[X]$ is the quantity:

$$\mathrm{Var}[X] = \mathrm{E}[(X - \mathrm{E}[X])^2]$$

The variance is a measure of dispersion for the distribution of X. A scale–free dispersion measure is the *coefficient of variation* CoV, defined through:

$$CoV^2 = \frac{\mathrm{Var}[X]}{(\mathrm{E}[X])^2}$$

Let $\overline{\mathbb{R}}^2$ denote the extended real plane $\overline{\mathbb{R}} \times \overline{\mathbb{R}}$. A 2–place real function H is a function whose domain $DomH$ is a subset of $\overline{\mathbb{R}}^2$ and whose range $RanH$ is a subset of $\mathbb{R}$. A rectangle R in $\overline{\mathbb{R}}^2$ is the cartesian product of two closed intervals, $R = [x_1, x_2] \times [y_1, y_2]$. Then, the vertices of the rectangle R are the four points $(x_i, y_j), i, j = 1, 2$. The unit square I^2 is the product $I \times I$ where I is the unit interval $I = [0, 1]$. A 2–place real function H is called 2–increasing when for all rectangles $R = [x_1, x_2] \times [y_1, y_2]$ whose vertices lie in $DomH$, $V_H(R) = H(x_2, y_2) - H(x_2, y_1) - H(x_1, y_2) + H(x_1, y_1) \geq 0$. A joint probability distribution function is defined as a 2–increasing function H with domain $\overline{\mathbb{R}}^2$ such that $H(x, -\infty) = 0$, $H(-\infty, y) = 0$ and $H(+\infty, +\infty) = 1$. The margins of a joint distribution function H are the functions $F_X(x) = H(x, +\infty)$ and $F_Y(y) = H(+\infty, y)$.

Appendix B

Monte Carlo methods

Monte Carlo methods are families of computational algorithms relying on repeated random sampling. These methods are used when deterministic calculus is not possible or practicable. A Monte Carlo simulation (MCS) aims to create samples of the input's PDF's. These samples need to represent the full range of the PDF's, with emphasing high probability areas more than low ones. The most common technique to achieve these requirements is to transform a set of pseudo–random numbers of $(0, 1)$ through the inverted CDF's. In short:

1. A random number u_i is drawn from $(0, 1)$.
2. The number u_i is transformed through probabilistic inversion, by means of the inverse CDF F_i^{-1}.

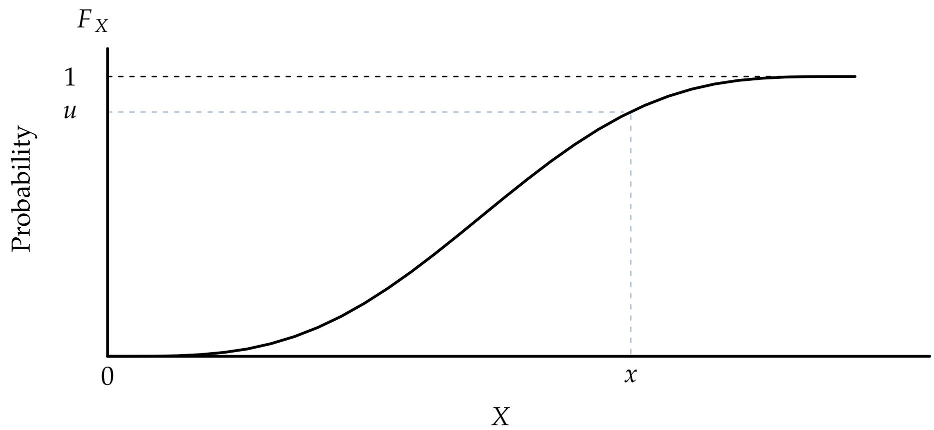

Figure B.1: The probabilistic inversion for simulating a random variable X.

In order to achieve the desired approximation with less iterations, stratified sampling methods can be used. For instance, in Latin Hypercube Sampling (LHS) the range of uncertain inputs is divided into equiprobable non–overlapping intervals; then, a random number (e.g. the midpoint) is drawn from each interval. LHS has recently appeared also in the field of cost engineering in construction [CYC09], [FPR09]. Instead of stratified sampling, the quasi–Monte Carlo methods can be used.

The Monte Carlo approach offers several benefits [KI07]: flexibility in incorporating uncertainty, computational simplification, software implementations that can be used by non–experts, handling of extreme cases, and improved accuracy with the technical advancement of computers. The major limitation of Monte Carlo is the tendency to blindly trust the output of an obscure computer process without checking possibly erroneous input (the so–called “Garbage In, Gospel Out” syndrom), which is obviously true for any complex model relying on “black box” numerical process.

Appendix C

Copulas

Copulas are special type of functions, introduced in 1959 by Abe Sklar. They were firstly used in the development of probabilistic metric spaces, and later in defining non–parametric dependence in applied statistics and uncertainty modelling. They are widely used today in several disciplines and applications, thanks to their ability to deal with arbitrary dependence. The main reason for the widespread use of copulas, is the usefulness in financial applications [Emb09]. The present section follows the classical monograph of Nelsen [Nel06].

Let a pair of random variables X and Y, with distribution functions F and G respectively, and a joint distribution function H. Each pair of real numbers (x, y) is associated with three numbers, $F(x)$, $G(y)$ and $H(x, y)$ which all lie in the interval $[0, 1]$. This way, each pair (x, y) of real numbers leads to a point $(F(x), G(y))$ in the unit square $[0, 1] \times [0, 1]$, and this ordered pair in turn corresponds to a number $H(x, y)$ in $[0, 1]$. This correspondence, which assigns the value of the joint distribution function $H(x, y)$ to each ordered pair of values of the individual distribution functions $(F(x), G(y))$, is a function, called *copula* of X and Y. In short, a copula is a distribution function of a random vector whose margins are uniformly distributed.

Formally, a copula is a function $C : [0, 1]^2 \to [0, 1]$ satisfying the properties:

1. $C(0, v) = C(u, 0) = 0, C(1, v) = v, C(u, 1) = u.$

2. $C(u, v)$ is increasing in u, v.

3. $C(u_2, v_2) - C(u_2, v_1) - C(u_1, v_2) + C(u_1, v_1) \geq 0$ for all u_1, u_2, v_1, v_2 in $[0, 1]$ (monotonicity condition).

Let C be a copula. For any $v \in [0, 1]$, the partial derivative $\partial C(u, v)/\partial u$ exists for almost all u, and for such v and u it holds that $0 \leq \partial C(u, v)/\partial u \leq 1$. Furtheremore, the function $u \to \partial C(u, v)/\partial u$ is defined and is nondecreasing almost everywhere on $[0, 1]$. The term "almost" is used in the sense of Lebesgue measure.

The cental result in the theory of copula functions is the following proposition:

Sklar's theorem. Let H be a joint cumulative distribution function with margins F, G. Then, there exists a copula C such that for all x, y:

$$H(x, y) = C(F(x), G(y)) \tag{C.1}$$

If F, G are continuous, then C is unique; otherwise C is uniquely determined on the range of $F \times G$. Conversely, if F, G are CDF's and C is a copula, then $H(x, y) = C(F(x), G(y))$ defines a joint CDF with margins F, G.

Equation (C.1) gives an expression for a joint distribution functions in terms of a copula and two univariate distribution functions. The equation can be inverted to express copulas in terms of a joint distribution function and the inverses[1] of the two margins:

$$C(u, v) = H(F^{-1}(u), G^{-1}(v))$$

When F and G are continuous, the latter formula provides a method of constructing copulas from joint distribution functions.

A few characteristic copula families can be mentioned:

Product copula

It describes the dependence structure between independent variables, as expressed with different notation by Equation (4.4):

$$C(u, v) = u \cdot v$$

Frank's copula

It is defined as:

$$C(u, v) = -\frac{1}{\theta}\left(1 + \frac{(e^{-\theta u} - 1)(e^{-\theta v} - 1)}{e^{-\theta} - 1}\right), \theta \neq 0$$

For $\theta < 0$ the Frank copula describes negative dependence; for $\theta > 0$ positive dependence. For $\theta \to -\infty$ and $\theta \to +\infty$ Frank's copula describes perfect negative

[1] If a margin is not strictly increasing, then it does not possess an inverse in the usual sense and so the generalised inverse is used instead.

and perfect positive dependence, respectively. It is a flexible one–parameter family, but the extension to higher dimensions is not straightforward.

Gaussian copula

It is defined as:

$$C(u,v) = \Phi_r(\Phi^{-1}(u), \Phi^{-1}(v))$$

where Φ_r is the CDF of the bivariate standard normal distribution with Pearson's correlation coefficient r. The simulation and practical use of this family in arbitrary dimensions is demonstrated in Section (4.8). The Gaussian copula does not have a closed form, but is calculated through double integration:

$$C_r(u,v) = \int_{-\infty}^{\Phi^{-1}(u)} \int_{-\infty}^{\Phi^{-1}(v)} \frac{1}{2\pi\sqrt{1-r^2}} \exp\left\{ \frac{-(s^2 - 2rst + t^2)}{2(1-r^2)} \right\} \mathrm{d}s\mathrm{d}t, r \neq -1, 0, 1$$

An extension of the Gaussian copula, with fast growing usage, is the t–copula, which contains an additional parameter, the degrees of freedom. This parameter allows for modelling tail dependence [DM05].

If X and Y are continuous random variables with copula C_{XY} and f, g are two strictly increasing functions defined on the range of $F \times G$, then it can be proved that $C_{f(X)g(Y)} = C_{XY}$. The following two relations reveal why Kendall's tau and Spearman's rho remain invariant (up to sign) under strictly monotone transformations of the margins, which is not true for Pearson's correlation coefficient.

$$\tau = 4 \iint\limits_{[0,1]^2} C(u,v)\mathrm{d}C(u,v) - 1$$

$$\rho = 12 \iint\limits_{[0,1]^2} C(u,v)\mathrm{d}u\mathrm{d}v - 3$$

The process of applying a copula methodology for simulating the response of a model relying on dependent variables can be outlined as follows:

1. The marginal distributions are specified.
2. The dependence structure is identified.
3. An appropriate copula family is selected.
4. A Monte Carlo simulation procedure is applied.

5. Realisations of the random vector are generated by probabilistic inversion.

6. The probability distribution of the desired stochastic model is constructed.

Several advantages of copula functions can be pronounced: they can capture non–linear dependence, they are very flexible (parametric, semi–parametric or non–parametric), any marginal distributions can be specified, the parametric families allow for sensitivity analysis. A few limitations[2] include the fact that extensions of bivariate to multivariate copulas is not straightforward, and that there is no obvious way to select the proper copula [Emb09].

[2]The uncritical use of copulas over classical stochastic calculus methods has been criticised [Mik06].

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